FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Mathematics Grades 10 - 12

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Chapter 17

Exponents - Grade 11

17.1 Introduction

In Grade 10 we studied exponential numbers and learnt that there were six laws that made working with exponential numbers easier. There is one law that we did not study in Grade 10. This will be described here.

17.2 Laws of Exponents

In Grade 10, we worked only with indices that were integers. What happens when the index is not an integer, but is a rational number? This leads us to the final law of exponents,

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \tag{17.1}$$

17.2.1 Exponential Law 7: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

We say that x is an nth root of b if $x^n = b$. For example, $(-1)^4 = 1$, so -1 is a 4th root of 1. Using law 6, we notice that

$$(a^{\frac{m}{n}})^n = a^{\frac{m}{n} \times n} = a^m \tag{17.2}$$

therefore $a^{\frac{m}{n}}$ must be an nth root of a^m. We can therefore say

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \tag{17.3}$$

where $\sqrt[n]{a^m}$ is the nth root of a^m (if it exists).

For example,

$$2^{\frac{2}{3}} = \sqrt[3]{2^2}$$

A number may not always have a real nth root. For example, if $n = 2$ and $a = -1$, then there is no real number such that $x^2 = -1$ because x^2 can never be a negative number.

Extension: Complex Numbers

There are numbers which can solve problems like $x^2 = -1$, but they are beyond the scope of this book. They are called *complex numbers*.

It is also possible for more than one nth root of a number to exist. For example, $(-2)^2 = 4$ and $2^2 = 4$, so both -2 and 2 are 2nd (square) roots of 4. Usually if there is more than one root, we choose the positive real solution and move on.

Worked Example 81: Rational Exponents

Question: Simplify without using a calculator:

$$\left(\frac{5}{4^{-1} - 9^{-1}}\right)^{\frac{1}{2}}$$

Answer

Step 1 : Rewrite negative exponents as numbers with postive indices

$$= \left(\frac{5}{\frac{1}{4} - \frac{1}{9}}\right)^{\frac{1}{2}}$$

Step 2 : Simplify inside brackets

$$\begin{aligned} &= \left(\frac{5}{1} \div \frac{9-4}{36}\right)^{\frac{1}{2}} \\ &= \left(\frac{5}{1} \times \frac{36}{5}\right)^{\frac{1}{2}} \\ &= (6^2)^{\frac{1}{2}} \end{aligned}$$

Step 3 : Apply exponential law 6

$$= 6$$

Worked Example 82: More rational Exponents

Question: Simplify:

$$(16x^4)^{\frac{3}{4}}$$

Answer

Step 1 : Covert the number co-efficient to index-form with a prime base

$$= (2^4 x^4)^{\frac{3}{4}}$$

Step 2 : Apply exponential laws

$$\begin{aligned} &= 2^{4\times\frac{3}{4}}.x^{4\times\frac{3}{4}} \\ &= 2^3.x^3 \\ &= 8x^3 \end{aligned}$$

Exercise: Applying laws

Use all the laws to:

1. Simplify:

   (a) $(x^0) + 5x^0 - (0{,}25)^{-0{,}5} + 8^{\frac{2}{3}}$
   (b) $s^{\frac{1}{2}} \div s^{\frac{1}{3}}$
   (c) $\frac{12m^{\frac{7}{9}}}{8m^{-\frac{11}{9}}}$
   (d) $(64m^6)^{\frac{2}{3}}$

2. Re-write the expression as a power of x:

$$x\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}$$

17.3 Exponentials in the Real-World

In Chapter 8, you used exponentials to calculate different types of interest, for example on a savings account or on a loan and compound growth.

Worked Example 83: Exponentials in the Real world

Question: A type of bacteria has a very high exponential growth rate at 80% every hour. If there are 10 bacteria, determine how many there will be in 5 hours, in 1 day and in 1 week?

Answer

Step 1 : $Population = Initial\ population \times (1 + growth\ percentage)^{time\ period\ in\ hours}$

Therefore, in this case:

$Population = 10(1{,}8)^n$, where n = number of hours

Step 2 : In 5 hours

$Population = 10(1{,}8)^5 = 188$

Step 3 : In 1 day = 24 hours

$Population = 10(1{,}8)^{24} = 13\ 382\ 588$

Step 4 : in 1 week = 168 hours

$Population = 10(1{,}8)^{168} = 7{,}687 \times 10^{43}$

Note this answer is given in scientific notation as it is a very big number.

Worked Example 84: More Exponentials in the Real world

Question: A species of extremely rare, deep water fish has an extremely long lifespan and rarely have children. If there are a total 821 of this type of fish and their growth rate is 2% each month, how many will there be in half of a year? What will be the population be in 10 years and in 100 years ?

Answer

Step 1 : $Population = Initial\ population \times (1 + growth\ percentage)^{time\ period\ in\ months}$

Therefore, in this case:
$Population = 821(1{,}02)^n$, where $n =$ number of months
Step 2 : In half a year = 6 months
$Population = 821(1{,}02)^6 = 924$
Step 3 : In 10 years = 120 months
$Population = 821(1{,}02)^{120} = 8\ 838$
Step 4 : in 100 years = 1 200 months
$Population = 821(1{,}02)^{1\ 200} = 1{,}716 \times 10^{13}$
Note this answer is also given in scientific notation as it is a very big number.

17.4 End of chapter Exercises

1. Simplify as far as possible:

   A $8^{-\frac{2}{3}}$
   B $\sqrt{16} + 8^{-\frac{2}{3}}$

2. Simplify:

   (a) $(x^3)^{\frac{4}{3}}$ (b) $(s^2)^{\frac{1}{2}}$
   (c) $(m^5)^{\frac{5}{3}}$ (d) $(-m^2)^{\frac{4}{3}}$
   (e) $-(m^2)^{\frac{4}{3}}$ (f) $(3y^{\frac{4}{3}})^4$

3. Simplify as much as you can:
$$\frac{3a^{-2}b^{15}c^{-5}}{(a^{-4}b^3c)^{\frac{-5}{2}}}$$

4. Simplify as much as you can:
$$\left(9a^6b^4\right)^{\frac{1}{2}}$$

5. Simplify as much as you can:
$$\left(a^{\frac{3}{2}}b^{\frac{3}{4}}\right)^{16}$$

6. Simplify:
$$x^3\sqrt{x}$$

7. Simplify:
$$\sqrt[3]{x^4b^5}$$

8. Re-write the expression as a power of x:
$$\frac{x\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}}{\sqrt[3]{x}}$$

Appendix A

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