Package 'cyclotomic'

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Type Package

Title The Field of Cyclotomic Numbers

Version 1.3.0

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Description The cyclotomic numbers are complex numbers that can be thought of as the rational numbers extended with the roots of unity. They are represented exactly, enabling exact computations. They contain the Gaussian rationals (complex numbers with rational real and imaginary parts) as well as the square roots of all rational numbers. They also contain the sine and cosine of all rational multiples of pi. The algorithms implemented in this package are taken from the 'Haskell' package 'cyclotomic', whose algorithms are adapted from code by Martin Schoenert and Thomas Breuer in the 'GAP' project (<https://www.gap-system.org/>). Cyclotomic numbers have applications in number theory, algebraic geometry, algebraic number theory, coding theory, and in the theory of graphs and combinatorics. They have connections to the theory of modular functions and modular curves.

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URL https://github.com/stla/cyclotomic

BugReports https://github.com/stla/cyclotomic/issues

Imports intmap, gmp, maybe, memoise, methods, numbers, VeryLargeIntegers

Encoding UTF-8

RoxygenNote 7.2.3

Collate 'Cyclotomic.R' 'arithmetic.R' 'conjugate.R' 'imports.R' 'is.R' 'maputils.R' 'mkCyclotomic.R' 'polar.R' 'rational.R' 'showCyclotomic.R' 'sqrt.R' 'trigonometry.R' 'zzz.R'

Suggests testthat (>= 3.0.0)

Config/testthat/edition 3

NeedsCompilation no

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Repository CRAN

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R topics documented:

`as.cyclotomic` *Coercion to a 'cyclotomic' object*

Description

Coercion to a 'cyclotomic' object

Usage

```
## S4 method for signature 'character'
as.cyclotomic(x)

## S4 method for signature 'cyclotomic'
as.cyclotomic(x)

## S4 method for signature 'numeric'
as.cyclotomic(x)

## S4 method for signature 'bigz'
as.cyclotomic(x)
```

```
## S4 method for signature 'bigq'
as.cyclotomic(x)
```

Arguments

x	a cyclotomic object or an object yielding a quoted integer or a quoted fraction after an application of as.character

Value

A cyclotomic object.

Examples

```
as.cyclotomic(2)
as.cyclotomic("1/3")
```

---

asComplex *Convert cyclotomic number to complex number*

Description

Convert a cyclotomic number to a complex number.

Usage

```
asComplex(cyc)
```

Arguments

cyc	a cyclotomic number

Value

A complex number (generally inexact).

Examples

```
asComplex(zeta(4))
```

conjugate *Conjugate cyclotomic number*

Description

Complex conjugate of a cyclotomic number.

Usage

```
conjugate(cyc)
```

Arguments

cyc	a cyclotomic number

Value

A cyclotomic number, the complex conjugate of cyc.

Examples

```
conjugate(zeta(4)) # should be -zeta(4)
```

cyclotomic-imports *Extract value from a 'Just' value*

Description

The from_just function is imported from the **maybe** package. Follow the link to its documentation: from_just. It has been imported for convenient use of the maybeRational function, which possibly returns a 'Just' value.

cyclotomic-unary *Unary operators for cyclotomic objects*

Description

Unary operators for cyclotomic objects.

Usage

```
## S4 method for signature 'cyclotomic,missing'
e1 + e2

## S4 method for signature 'cyclotomic,missing'
e1 - e2
```

Arguments

e1	object of class cyclotomic
e2	nothing

Value

A cyclotomic object.

cycSqrt *Square root as a cyclotomic number*

Description

Square root of an integer or a rational number as a cyclotomic number. This is slow.

Usage

```
cycSqrt(x)
```

Arguments

x	an integer, a **gmp** rational number (bigq object), or a fraction given as a string (e.g. "5/3")

Value

The square root of x as a cyclotomic number.

Examples

```
cycSqrt(2)
phi <- (1 + cycSqrt(5)) / 2 # the golden ratio
phi^2 - phi # should be 1
```

`imaginaryPart` *Imaginary part of cyclotomic number.*

Description

The imaginary part of a cyclotomic number.

Usage

```
imaginaryPart(cyc)
```

Arguments

`cyc`	a cyclotomic number

Value

A cyclotomic number.

Examples

```
imaginaryPart(zeta(9))
```

`isGaussianRational` *Is the cyclotomic a Gaussian rational?*

Description

Checks whether a cyclotomic number is a Gaussian rational number.

Usage

```
isGaussianRational(cyc)
```

Arguments

`cyc`	a cyclotomic number

Value

A Boolean value.

isRational *Is the cyclotomic a rational number?*

Description

Checks whether a cyclotomic number is a rational number.

Usage

```
isRational(cyc)
```

Arguments

cyc	a cyclotomic number

Value

A Boolean value.

See Also

maybeRational

isReal *Is the cyclotomic a real number?*

Description

Checks whether a cyclotomic number is a real number.

Usage

```
isReal(cyc)
```

Arguments

cyc	a cyclotomic number

Value

A Boolean value.

maybeRational *Cyclotomic as exact rational number if possible*

Description

Cyclotomic number as exact rational number if possible.

Usage

```
maybeRational(cyc)
```

Arguments

cyc	a cyclotomic number

Value

A maybe value, just a rational number if cyc is a rational number, nothing otherwise.

See Also

isRational

Examples

```
maybeRational(zeta(4))
maybeRational(cosDeg(60)) # use `from_just` to get the value
```

polar *Polar complex number with rational magnitude and angle*

Description

Complex number in polar form with rational magnitude and rational angle as a cyclotomic number.

Usage

```
polarDeg(r, theta)

polarRev(r, theta)
```

Arguments

r	magnitude, an integer number, a **gmp** rational number, or a fraction given as a character string (e.g. "2/7")
theta	angle, an integer number, a **gmp** rational number, or a fraction given as a character string (e.g. "2/7"); for polarDeg the angle is given in degrees and for polarRev it is given in revolutions

Value

A cyclotomic number.

Examples

```
polarDeg(1, 90)    # should be zeta(4)
polarRev(1, "1/4") # should be zeta(4) as well
```

---

quadraticRoots *Roots of quadratic polynomial*

Description

Roots of a polynomial of degree 2 as cyclotomic numbers.

Usage

```
quadraticRoots(a, b, c)
```

Arguments

a, b, c	the coefficients of the polynomial

Value

A list of two cyclotomic numbers, the roots of the polynomial $ax^2 + bx + c$.

Examples

```
library(cyclotomic)
quadraticRoots(a = 1, b = 2, c = -1)
```

---

realPart *Real part of cyclotomic number.*

Description

The real part of a cyclotomic number.

Usage

```
realPart(cyc)
```

Arguments

cyc	a cyclotomic number

Value

A cyclotomic number.

Examples

```
realPart(zeta(9))
```

`trigonometry` *Cosine and sine of a rational number*

Description

Cosine and sine of a rational angle as a cyclotomic number.

Usage

```
cosDeg(theta)

sinDeg(theta)

cosRev(theta)

sinRev(theta)
```

Arguments

`theta`	an integer number, a **gmp** rational number, or a fraction given as a character string (e.g. `"2/7"`)

Details

The function `cosDeg`, resp. `sinDeg`, returns the cosine, resp. the sine, of its argument assumed to be given in degrees. The function `cosRev`, resp. `sinRev`, returns the cosine, resp. the sine, of its argument assumed to be given in revolutions.

Value

A cyclotomic number.

Examples

```
cosDeg(60)
cosDeg("2/3")^2 + sinDeg("2/3")^2 == 1
```

`zeta` *The primitive n-th root of unity.*

Description

For example, ‘zeta(4) = i‘ is the primitive 4th root of unity, and ‘zeta(5) = exp(2*pi*i/5)‘ is the primitive 5th root of unity. In general, ‘zeta(n) = exp(2*pi*i/n)‘.

Usage

```
zeta(n)
```

Arguments

`n`	a positive integer

Value

A cyclotomic number.

Examples

```
zeta(4)
```

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