

# Package ‘bayespm’

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**Title** Bayesian Statistical Process Monitoring

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**Description** The R-package bayespm implements Bayesian Statistical Process Control and Monitoring (SPC/M) methodology. These methods utilize available prior information and/or historical data, providing efficient online quality monitoring of a process, in terms of identifying moderate/large transient shifts (i.e., outliers) or persistent shifts of medium/small size in the process. These self-starting, sequentially updated tools can also run under complete absence of any prior information. The Predictive Control Charts (PCC) are introduced for the quality monitoring of data from any discrete or continuous distribution that is a member of the regular exponential family. The Predictive Ratio CUSUMs (PRC) are introduced for the Binomial, Poisson and Normal data (a later version of the library will cover all the remaining distributions from the regular exponential family). The PCC targets transient process shifts of typically large size (a.k.a. outliers), while PRC is focused in detecting persistent (structural) shifts that might be of medium or even small size. Apart from monitoring, both PCC and PRC provide the sequentially updated posterior inference for the monitored parameter. Bourazas K., Kiagias D. and Tsiamyrtzis P. (2022) “Predictive Control Charts (PCC): A Bayesian approach in online monitoring of short runs” <doi:10.1080/00224065.2021.1916413>, Bourazas K., Sobas F. and Tsiamyrtzis, P. 2023. “Predictive ratio CUSUM (PRC): A Bayesian approach in online change point detection of short runs” <doi:10.1080/00224065.2022.2161434>, Bourazas K., Sobas F. and Tsiamyrtzis, P. 2023. “Design and properties of the predictive ratio cusum (PRC) control charts” <doi:10.1080/00224065.2022.2161435>.

**License** GPL (>= 2)

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rmutl, invgamma

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## R topics documented:

aPTT	2
betabinom_HM	3
betanbinom_HM	4
binom_PCC	5
binom_PRC	7
binom_PRC_h	10
compgamma_HD	12
ECE	13
gamma_PCC	14
gb2_HD	16
invgamma_PCC	17
lnorm1_PCC	19
lnorm2_PCC	21
lnorm3_PCC	23
lnorm_HD	26
lt_HD	27
nbinom_HM	28
nbinom_PCC	29
norm1_PCC	31
norm2_PCC	33
norm3_PCC	35
norm_HD	38
norm_mean2_PRC	39
norm_mean2_PRC_h	42
pois_PCC	43
pois_PRC	46
pois_PRC_h	48
t_HD	49
<b>Index</b>	<b>51</b>

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aPTT

*Dataset for PCC process for Normal with both parameters unknown*

---

### Description

The activated Partial Thromboplastin Time (aPTT) dataset is a data frame that contains two columns. The first column (aPTT\_current) represents 30 sequentially gathered current observations from the daily routine of a medical laboratory. The values refer to the activated Partial Thromboplastin Time (aPTT), measured in seconds. The second column (aPTT\_historical) refers to 30 historical In Control measurements of the same variable. APTT is a blood test that characterizes coagulation of the blood. More specifically, it is the time that until the coagulation when reagents are added to plasma.

**Usage**

```
data("aPTT")
```

**Examples**

```
# Loading data
attach(aPTT)

# Plotting data
plot( 1:length(aPTT_current), aPTT_current, type = "b", lty = 1, xlab = "", ylab = "",
      ylim = c(27.3, 33.4), xlim = c(-30, 30), lwd = 1, pch = 16, axes = FALSE, yaxs = "i",
      main = "aPTT dataset" )

# x - axis for historical and current data
pastx <- c( -30, -20, -10, 0 ) ; currentx <- c( 0, 10, 20, 30 )
# Adding points
points( -length(aPTT_historical):(-1), aPTT_historical,
        type = "b", lty = 2, xlab = "", ylab = "", lwd = 1, pch = 21, col = "gray55" )
# Adding axis with names
axis(2)
mtext( "Current Data", side = 1, at = 15, line = 2.2, cex = 1.1 )
mtext( "Historical Data", side = 1, at = -15, line = 2.2, cex = 1.1, col = "gray55" )
mtext( "aPTT [sec]", side = 2, line = 2.2, cex = 1.1 )
axis( 1, at = currentx, labels = currentx )
axis( 1, at = pastx, labels = pastx, col.axis = "gray55", col = "gray55", lty = 2 )
segments( 0, 27.5, 0, 33.5, lwd = 1 )
```

---

betabinom\_HM

*The Highest Mass (HM) interval of Beta-Binomial distribution.*


---

**Description**

betabinom\_HM is used to derive the narrowest interval of a predetermined coverage from a Beta-Binomial distribution, containing the Highest Mass (HM) values.

**Usage**

```
betabinom_HM( cover = NULL, n = NULL, a = NULL, b = NULL, plot = FALSE,
              xlab = "x", ylab = "Probability" )
```

**Arguments**

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Mass (HM).
n	scalar (integer); It is the number of trials of the Binomial distribution and needs to be a number.
a	scalar (positive); It is the shape1 parameter of the Beta distribution and needs to be a number.

b	scalar (positive); It is the shape2 parameter of the Beta distribution and needs to be a number.
plot	logical; if TRUE, the plot of the HM region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HM plot. The default values are "x" and "Probability".

### Details

betabinom\_HM provides the Highest Mass (HM) of a Beta-Binomial distribution. HM is the narrowest region, with the minimum absolute difference from a predetermined coverage. The values included in HM have probability larger than every point outside the region.

### Value

Returns a data frame with the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

### Examples

```
betabinom_HM(0.95, 10, 20, 180, plot = TRUE)
```

---

betabinom_HM	<i>The Highest Mass (HM) interval of Beta-Negative Binomial distribution.</i>
--------------	---

---

### Description

betabinom\_HM is used to derive the narrowest interval of a predetermined coverage from a Beta-Negative Binomial distribution, containing the Highest Mass (HM) values.

### Usage

```
betabinom_HM( cover = NULL, r = NULL, a = NULL, b = NULL, plot = FALSE,
              xlab = "x", ylab = "Probability" )
```

### Arguments

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Mass (HM).
r	scalar (integer); It is the number of failures of the Negative Binomial distribution and needs to be a number.
a	scalar (positive); It is the shape1 parameter of the Beta distribution and needs to be a number.
b	scalar (positive); It is the shape2 parameter of the Beta distribution and needs to be a number.
plot	logical; if TRUE, the plot of the HM region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HM plot. The default values are "x" and "Probability".

**Details**

betanbinom\_HM provides the Highest Mass (HM) of a Beta-Negative Binomial distribution. HM is the narrowest region, with the minimum absolute difference from a pretermined coverage. The values included in HM have probability larger than every point outside the region.

**Value**

Returns a data frame with the upper and the lower limits of the Highest Predictive Denisty or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

**Examples**

```
betanbinom_HM(0.95, 5, 20, 80, plot = TRUE)
```

---

 binom\_PCC

*PCC for Binomial data with probability parameter unknown*


---

**Description**

binom\_PCC is used to derive the Predictive Control Chart (PCC) for Binomial data, when the probability parameter is unknown and the trials are known.

**Usage**

```
binom_PCC( data = NULL, n = NULL, historical_data = NULL,
           historical_n = NULL, a0 = 1/2, b0 = 1/2, alpha_0 = NULL,
           ARL_0 = 370.4, FAP = NULL, FIR = FALSE, ffir = .99, afir = 1/8,
           summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
           historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
           xlab = "Observations", ylab = "Quality characteristic Values",
           main = "PCC Binomial with unknown probability" )
```

**Arguments**

data	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
n	vector; number of trials for given dataset. It needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
historical_n	vector; number of trials for given historical dataset. It needs to be in a vector form.
a0	scalar (positive); shape hyperparameter of the prior Beta(a, b). It needs to be a number. The default is 1/2 and it refers to the initial reference prior Beta(1/2, 1/2).

b0	scalar (positive); shape hyperparameter of the prior Beta(a, b). It needs to be a number. The default is 1/2 and it refers to the initial reference prior Beta(1/2, 1/2).
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is 1/n_0, where n_0 is the size of the historical data.
ARL_0	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
FAP	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of ARL_0. It is based on the Sidak's correction.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
fFIR	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
aFIR	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC Binomial with unknown probability" respectively

## Details

binom\_PCC provides the Predictive Control Chart (PCC) process for Binomial data, when the probability parameter is unknown and the trials are known. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown probability is a Beta:  $\text{Beta}(a_0, b_0)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $\text{Beta}(1/2, 1/2)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either  $\text{ARL}_0$  or FAP.

## Value

Runs the PCC process, while if  $\text{FIR}=\text{T}$ , then the FIR-PCC is implemented. If  $\text{plot}=\text{T}$ , then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPd/M) region, and an indication in case of alarms. If  $\text{PCC\_PLOT}=\text{T}$ , then the PCC process is visualized, while if  $\text{historical\_data\_PLOT}=\text{T}$ , the historical data are added to the plot. If  $\text{Prior-Posterior\_PLOT} = \text{T}$ , a plot with the prior and the posterior is provided. Finally, if  $\text{pdf\_report}=\text{T}$ , all the output is reported and saved into a pdf file.

## Examples

```
# 30 Binomial observations introducing an outlier at the 15th observation
set.seed(10)
SimData <- rbinom( n = 30, size = 20, prob = 0.6 )
SimData[15] <- round( SimData[15] + 3*sqrt(20*0.6*0.4) )
binom_PCC( SimData, n = rep(20, 30) )
```

---

binom\_PRC

*PRC for Binomial data with probability parameter unknown*

---

## Description

binom\_PRC is used to derive the Predictive Ratio CUSUM (PRC) for Binomial data, when the probability parameter is unknown.

## Usage

```
binom_PRC( data = NULL, n = NULL, historical_data = NULL, historical_n = NULL,
  a0 = 1/2, b0 = 1/2, alpha_0 = NULL, k=2, two.sided=FALSE,
  h = log(100), FIR = FALSE, fFIR = 1/2, dFIR = 3/4,
  summary_list = TRUE, PRC_PLOT = TRUE, pdf_report = FALSE,
  path_pdf_report = tempdir(),
  xlab = "Observation Order", ylab = "PRC cumulative statistics",
  main = "PRC Binomial with unknown probability" )
```

**Arguments**

data	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
n	vector; number of trials for given dataset. It needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
historical_n	vector; number of trials for given historical dataset. It needs to be in a vector form.
a0	scalar (positive); shape hyperparameter of the prior Beta(a, b). It needs to be a number. The default is 1/2 and it refers to the initial reference prior Beta(1/2, 1/2).
b0	scalar (positive); shape hyperparameter of the prior Beta(a, b). It needs to be a number. The default is 1/2 and it refers to the initial reference prior Beta(1/2, 1/2).
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is 1/n_0, where n_0 is the size of the historical data.
k	scalar (positive); tuning parameter, which represents deviation between IC and OOC state. Precisely, if $k > 1$ then the OOC state corresponds to a $(k-1)100\%$ increase in the expected odds of the success probability $p$ and if $k < 1$ corresponds $(1-k)100\%$ decrease. The default value is 2.
two.sided	logical; If TRUE, then a two sided PRC is employed with tuning parameters $k$ and $1/k$ respectively. It is FALSE by default.
h	scalar (positive); Decision limit and we raise an alarm if the cumulative statistic exceeds it (or $2h$ for two sided shifts). The default value is $\log(100)=4.605$ , for which we have a decisive cumulative evidence in favor of the OOC state.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PRC is applied, which is an exponentially decreasing adjustment for the initial first tests, inflating the PRC statistics.
fFIR	non-negative number; It is used if FIR=TRUE. It represents the proportion of the inflation for the PRC statistic in the for the first test. The default value is 1/2.
dFIR	a number between 0 and 1; It is used if FIR=TRUE. It is a smoothing parameter, specifying the exponential decay of the adjustment (the smaller the $d$ the fastest the decay). The default value is 3/4.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PRC_PLOT	logical; if TRUE, the PRC plot is displayed. It is TRUE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().



xlab, ylab, main

The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "PRC cumulative statistics" and "PRC Binomial with unknown probability" respectively

## Details

binom\_PRC provides the Predictive Ratio CUSUM (PRC) process for Binomial data, when the probability parameter is unknown. The PRC process is based on sequential comparison between the IC and OOC state, via their predictive distributions. The parameter  $k$  represents the deviance between these two states. The default value is  $k=2$ , which corresponds to a doubling of the expected odds ratio of the probability of success. The testing starts as early as the second observation.

The initial prior for the unknown probability is a Beta:  $\text{Beta}(a, b)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $\text{Beta}(1/2, 1/2)$ , without the use of historical data.

A FIR option is available by inflating the first few PRC statistics. The choice of the decision limit  $h$  reflects the false alarm tolerance.

## Examples

```
# the PRC process for the first 30 data points in the third application in
# "Design and properties of the Predictive Ratio Cusum (PRC) control charts"

### HISTORICAL DATA (FIRST BATCH)
HD <- c( 3, 3, 1, 5, 2, 1, 3, 3, 3, 0, 2, 1, 2, 1, 4, 1, 1, 0, 3, 2, 4, 6, 0, 1, 3, 2, 2, 4, 2, 1 )

### Bernoulli trials
nn <- 50

N0 <- length(HD)
NN0 <- rep(50, N0)

binom_PRC(data = HD, n = NN0)

# the PRC process for the last 10 data points in the third application in
# "Design and properties of the Predictive Ratio Cusum (PRC) control charts"

### prior parameters before the first batch

a0 <- 1/2
b0 <- 1/2

### posterior parameters after the first batch

ap <- sum(HD) + a0
bp <- sum(NN0) - sum(HD) + b0
```

```

h1 = 4.332 # the decision limit is derived by the function binom_PRC_h

### CURRENT DATA (SECOND BATCH)
CD <- c(2, 4, 5, 2, 4, 8, 4, 4, 8, 5)

N <- length(CD)
NN <- rep(50, N)

binom_PRC(data = CD, n = NN, a0 = ap, b0 = bp, h = h1)

```

---

binom_PRC_h	<i>Derivation of the decision limit for the PRC for Binomial data with probability parameter unknown</i>
-------------	--

---

### Description

binom\_PRC\_h is used to estimate the decision limit  $h$  for the Predictive Ratio CUSUM (PRC) for Binomial data. The process controls the false alarms tolerance, providing a lower bound of ARL<sub>0</sub> or FAP, based on the marginal distribution.

### Usage

```

binom_PRC_h( ARL_0 = 370.4, FAP = NULL, N = NULL, n = NULL,
             historical_data = NULL, historical_n = NULL,
             a0 = NULL, b0 = NULL, alpha_0 = NULL, k = 2,
             it = 1e4, ARL0tol = 10/it )

```

### Arguments

ARL <sub>0</sub>	scalar (positive); lower bound of the In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4.
FAP	scalar (between 0 and 1); lower bound of the False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of $N$ hypothesis tests and it can be used instead of ARL <sub>0</sub> .
N	scalar (natural); number of $N$ hypothesis tests. It is used for the FAP determination.
n	vector; number of trials for given dataset. It needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
historical_n	vector; number of trials for given historical dataset. It needs to be in a vector form.
a0	scalar (positive); shape hyperparameter of the prior Beta(a, b). It needs to be a number.

b0	scalar (positive); shape hyperparameter of the prior Beta(a, b). It needs to be a number.
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is 1/n_0, where n_0 is the size of the historical data.
k	scalar (positive); tuning parameter, which represents deviation between IC and OOC state. Precisely, if k>1 then the OOC state corresponds to a (k-1)100% increase in the expected odds of the success probability p and if k<1 corresponds (1-k)100% decrease. The default value is 2.
it	scalar (natural); iterations of the simulation. The default value is 10000.
ARL0tol	scalar (positive); relative tolerance of ARL_0 estimate. It represents the maximum amount of error which is allowed. The default value is 10/it.

### Details

binom\_PRC\_h estimates the decision limit h regarding the desired upper bound of false alarms tolerance for the Predictive Ratio CUSUM (PRC) process for Binomial data, when the probability parameter is unknown. The estimate concerns an one-sided PRC process and the default setup corresponds to a doubling of the expected odds ratio of the probability of success (k=2). The estimate is based on the use of the marginal distribution (prior predictive) and the incorporation of historical data is possible via the power prior.

The default metric of the false alarms tolerance is ARL\_0=370.4, while a FAP option is available. The decision limit is obtained via the numerical method regula falsi when ARL\_0 is selected, while simulated samples of fixed size are used when FAP is of interest.

### Examples

```
binom_PRC_h(ARL_0 = NULL, FAP = 0.05, N = 20, n = 10, a0 = 20, b0 = 180)

# derivation of the decision limit of the third application in
# "Design and properties of the Predictive Ratio Cusum (PRC) control charts"

ar10 <- 400
ap <- 66.5
bp <- 1434.5
k1 <- 2

# To replicate results from application set 'ARL0tol = .001' and 'it = 1e4'
binom_PRC_h(ARL_0 = ar10, ARL0tol = .01, k = k1, n = 50, a0 = ap, b0 = bp, it = 1e3)
```

---

 compgamma\_HD

*The Highest Density (HD) interval of Compound Gamma distribution.*


---

### Description

compgamma\_HD is used to derive the narrowest interval of a predetermined coverage from a Compound Gamma distribution, containing the Highest Density (HD) values.

### Usage

```
compgamma_HD( cover = NULL, a = NULL, c = NULL, d = NULL, plot = FALSE,
              xlab = "x", ylab = "Density" )
```

### Arguments

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Density (HD).
a	scalar (positive); It is the shape1 parameter of the Compound Gamma distribution and needs to be a number.
c	scalar (positive); It is the shape2 parameter of the Compound Gamma distribution and needs to be a number.
d	scalar (positive); It is the scale parameter of the Compound Gamma distribution and needs to be a number.
plot	logical; if TRUE, the plot of the HD region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HD plot. The default values are "x" and "Density".

### Details

compgamma\_HD provides the Highest Density (HD) of a Compound Gamma distribution. HD is the narrowest interval with a predetermined coverage. The values included in HD have density larger than every point outside the region.

### Value

Returns a data frame with the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

### Examples

```
compgamma_HD(0.95, 2, 10, 10, plot = TRUE)
```

ECE

*ECE dataset for the PCC process for Poisson with rate parameter unknown*

## Description

The Electrical Complex Equipment (ECE) dataset represents 25 paired observations in a data frame. The first column (`defect_counts`) refers the number of defects per inspected number of units, encountered in a complex electrical equipment of an assembly line. The second column (`inspected_units`) contains the corresponding inspected number of units for the counting of defects. The data were also analyzed by Hansen and Ghare (1987) and Bayarri and Garcia-Donato (2005).

## Usage

```
data("ECE")
```

## References

[1] Bayarri, M. J., and Garcia-Donato, G. (2005), "A Bayesian Sequential Look at u-Control Charts", *Technometrics*, 47, 2, pp. 142-151

[2] Hansen, B., and Ghare, P. (1987), "Quality Control and Application, Prentice-Hall", Englewood Cliffs, NJ

## Examples

```
# Loading data
attach(ECE)

# Plotting data
graphpar <- par( oma = c(1,3,2,3) )
plot( 1:length(defect_counts), defect_counts/inspected_units, type = "b", lty = 1,
      xlab = "Observations", ylab = "", xlim = c(0, 25), ylim = c(1.5, 10.5),
      lwd = 1, pch = 16, axes = FALSE, yaxs = "i", main = "ECE dataset" )

# Adding points
points( 1:length(defect_counts), inspected_units, type = "b",
       lty = 2, lwd = 1, pch = 21, col = "gray55" )

# Adding legend
legend( "topleft", legend=c(expression(paste(s[i])), expression(paste(x[i]/s[i])) ), bty = "n",
       cex = 0.8, lty = c(2, 1), lwd = 1, col = c("gray55", "black"), pch = c(21, 16) )

# Adding axis with names
axis(1) ; axis(2) ; axis(4, col.axis = "gray55", col = "gray55")
mtext( "Number of Defects \n per unit", side = 2, line = 2.2, cex = 0.9 )
mtext( "Inspected units", side = 4, line = 2, cex = 0.9, col = "gray55" )

# Resetting graphical parameters
par(graphpar)
```

gamma\_PCC

*PCC for Gamma data with rate parameter unknown***Description**

gamma\_PCC is used to derive the Predictive Control Chart (PCC) for Gamma data, when the rate parameter is unknown and the shape parameter is known.

**Usage**

```
gamma_PCC( data = NULL, historical_data = NULL, a1 = NULL,
           c0 = 0, d0 = 0, alpha_0 = NULL,
           ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR = 1/8,
           summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
           historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
           xlab = "Observation Order", ylab = "Quality characteristic Values",
           main = "PCC Gamma with rate parameter unknown" )
```

**Arguments**

data	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
a1	scalar (positive); parameter of the likelihood. It is the known shape parameter of the data and needs to be a number.
c0	scalar (positive); scalar (positive); hyperparameter of the prior $G(c_0, d_0)$ . It needs to be a number. The default is 0 and it refers to the initial reference prior $G(0, 0)$ .
d0	scalar (positive); scalar (positive); hyperparameter of the prior $G(c_0, d_0)$ . It needs to be a number. The default is 0 and it refers to the initial reference prior $G(0, 0)$ .
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.
ARL_0	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
FAP	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of $N$ hypothesis tests and it can be used instead of $ARL_0$ . It is based on the Sidak's correction.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.

ffIR	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
aFIR	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC Gamma with rate parameter unknown" respectively.

## Details

gamma\_PCC provides the Predictive Control Chart (PCC) process for Poisson data, when the rate parameter is unknown and the shape parameter is known. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown parameters is a Gamma:  $G(c_0, d_0)$ , where  $d_0$  is the rate parameter. Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $G(0,0)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either ARL<sub>0</sub> or FAP.

## Value

Runs the PCC process, while if FIR=T, then the FIR-PCC is implemented. If plot=T, then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, and an indication in case of alarms. If PCC\_PLOT=T, then the PCC process

is visualized, while if `historical_data_PLOT=T`, the historical data are added to the plot. If `Prior-Posterior_PLOT = T`, a plot with the prior and the posterior is provided. Finally, if `pdf_report=T`, all the output is reported and saved into a pdf file.

### Examples

```
# 30 Gamma observations introducing an outlier of 3*sd at the 15th observation
set.seed(100)
out <- rgamma( n = 30, shape = 4, rate = 4 )
out[15] <- out[15] + 1.5
gamma_PCC( out, a1 = 4 )
```

---

gb2_HD	<i>The Highest Density (HD) interval of Generalized Beta of the second kind distribution.</i>
--------	---

---

### Description

gb2\_HD is used to derive the narrowest interval of a predetermined coverage from a Generalized Beta of the second kind distribution, containing the Highest Density (HD) values.

### Usage

```
gb2_HD( cover = NULL, a = NULL, c = NULL, d = NULL, plot = FALSE,
        xlab = "x", ylab = "Density" )
```

### Arguments

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Density (HD).
a	scalar (positive); It is the shape1 parameter of the Generalized Beta of the second kind distribution and needs to be a number.
c	scalar (positive); It is the shape2 parameter of the Generalized Beta of the second kind distribution and needs to be a number.
d	scalar (positive); It is the scale parameter of the Generalized Beta of the second kind distribution and needs to be a number.
plot	logical; if TRUE, the plot of the HD region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HD plot. The default values are "x" and "Density".

### Details

gb2\_HD provides the Highest Density (HD) of a Generalized Beta of the second kind distribution. HD is the narrowest interval with a pretermined coverage. The values included in HD have density larger than every point outside the region.



**Value**

Returns a data frame with the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

**Examples**

```
gb2_HD(0.95, 4, 6, 6, plot = TRUE)
```

---

 invgamma\_PCC

*PCC for Inverse-Gamma data with scale parameter unknown*


---

**Description**

invgamma\_PCC is used to derive the Predictive Control Chart (PCC) for Inverse-Gamma data, when the scale parameter is unknown and the shape parameter is known.

**Usage**

```
invgamma_PCC( data = NULL, historical_data = NULL,
              a1 = NULL, c0 = 0, d0 = 0, alpha_0 = NULL,
              ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR = 1/8,
              summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
              historical_data_PLOT = FALSE, pdf_report = FALSE,
              path_pdf_report = tempdir(),
              xlab = "Observation Order", ylab = "Quality characteristic Values",
              main = "PCC Inverse-Gamma with unknown scale parameter" )
```

**Arguments**

data	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
a1	scalar (positive); parameter of the likelihood. It is the known shape parameter of the data and needs to be a number.
c0	scalar (positive); scalar (positive); hyperparameter of the prior $G(c0, d0)$ . It needs to be a number. The default is 0 and it refers to the initial reference prior $G(0, 0)$ .
d0	scalar (positive); scalar (positive); hyperparameter of the prior $G(c0, d0)$ . It needs to be a number. The default is 0 and it refers to the initial reference prior $G(0, 0)$ .
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.

ARL_0	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
FAP	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of ARL_0. It is based on the Sidak's correction.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
ffFIR	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
aFIR	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC Inverse-Gamma with unknown scale parameter" respectively

## Details

invgamma\_PCC is used to derive the Predictive Control Chart (PCC) for Inverse-Gamma data, when the scale parameter is unknown and the shape parameter is known. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown scale is a Gamma:  $\text{Gamma}(c_0, d_0)$ . Furthermore, the direct use of historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $\text{Gamma}(0, 0)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either ARL\_0 or FAP.

**Value**

Runs the PCC process, while if FIR=T, then the FIR-PCC is implemented. If plot=T, then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, and an indication in case of alarms. If PCC\_PLOT=T, then the PCC process is visualized, while if historical\_data\_PLOT=T, the historical data are added to the plot. If Prior-Posterior\_PLOT = T, a plot with the prior and the posterior is provided. Finally, if pdf\_report=T, all the output is reported and saved into a pdf file.

**Examples**

```
# 30 Inverse-Gamma observations introducing an outlier at the 15th observation
set.seed(100)
SimData <- 1/rgamma(n = 30, shape = 3, rate = 2)
SimData[15] <- SimData[15] + 3
invgamma_PCC(SimData, al = 3)
```

Inorm1\_PCC

*PCC for LogNormal data with scale parameter unknown***Description**

Inorm1\_PCC is used to derive the Predictive Control Chart (PCC) for Normal data, when the scale parameter is unknown and the shape parameter is known.

**Usage**

```
Inorm1_PCC(data = NULL, historical_data = NULL,
            sd1 = NULL, mu0 = 0, sd0 = 10^6, alpha_0 = NULL,
            ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR = 1/8,
            summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
            historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
            xlab = "Observations", ylab = "Quality characteristic Values",
            main = "PCC LogNormal with unknown mean")
```

**Arguments**

data	vector; a dataset for PCC implementation. Data needs to be in a vector form.
historical_data	vector; an optional dataset of historical data. Historical data needs to be in a vector form.
sd1	scalar (positive); parameter of the likelihood. It is the known standard deviation of the data and needs to be a number.
mu0	scalar; hyperparameter of the prior $N(\mu_0, sd_0^2)$ . It needs to be a number. The default is 0 and it refers to the initial reference prior $N(0, \infty)$ .
sd0	scalar (positive); hyperparameter of the prior $N(\mu_0, sd_0^2)$ . It needs to be a number. The default is $10^6$ and it refers to the initial reference prior $N(0, \infty)$ .

alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.
ARL_0	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
FAP	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of $N$ hypothesis tests and it can be used instead of ARL_0. It is based on the Sidak's correction.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
fFIR	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
aFIR	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC LogNormal with unknown mean" respectively.

## Details

Inorm1\_PCC provides the Predictive Control Chart (PCC) process for LogNormal data, when the scale parameter is unknown and the shape parameter is known. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown scale parameter is a Normal:  $N(\mu_0, \text{sd}_0^2)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this

case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $\text{NIG}(0, \infty)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either  $\text{ARL}_0$  or FAP.

### Value

Runs the PCC process, while if  $\text{FIR}=\text{T}$ , then the FIR-PCC is implemented. If  $\text{plot}=\text{T}$ , then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, and an indication in case of alarms. If  $\text{PCC\_PLOT}=\text{T}$ , then the PCC process is visualized, while if  $\text{historical\_data\_PLOT}=\text{T}$ , the historical data are added to the plot. If  $\text{PriorPosterior\_PLOT}=\text{T}$ , a plot with the prior and the posterior is provided. Finally, if  $\text{pdf\_report}=\text{T}$ , all the output is reported and saved into a pdf file.

### Examples

```
set.seed(9)
SimData <- rlnorm(n = 30, meanlog = 0, sdlog = 1/2)
SimData[15] <- SimData[15] + 3*sqrt( exp(1/4)*( exp(1/4)-1 ) )
plot(SimData)
Inorm1_PCC(SimData, sdl = 1/2)
```

---

Inorm2\_PCC

*PCC for LogNormal data with shape parameter unknown*

---

### Description

Inorm2\_PCC is used to derive the Predictive Control Chart (PCC) for LogNormal data, when the shape parameter is unknown and the scale parameter is known.

### Usage

```
Inorm2_PCC(data = NULL, historical_data = NULL,
           m1 = NULL, a0 = 0, b0 = 0, alpha_0 = NULL,
           ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR = 1/8,
           summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
           historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
           xlab = "Observation Order", ylab = "Quality characteristic Values",
           main = "PCC LogNormal with unknown variance")
```

### Arguments

**data** vector; a dataset for PCC implementation. Data needs to be in a vector form.

**historical\_data** vector; an optional dataset of historical data. Historical data needs to be in a vector form.

<code>m1</code>	scalar; parameter of the likelihood. It is the known mean of the data and needs to be a number.
<code>a0</code>	scalar (positive); hyperparameter of the prior IG(a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior IG(0, 0).
<code>b0</code>	scalar (positive); hyperparameter of the prior IG(a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior IG(0, 0).
<code>alpha_0</code>	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.
<code>ARL_0</code>	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
<code>FAP</code>	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of <code>ARL_0</code> . It is based on the Sidak's correction.
<code>FIR</code>	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
<code>fFIR</code>	a number between 0 and 1; It is used if <code>FIR=TRUE</code> . The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
<code>aFIR</code>	non-negative number; It is used if <code>FIR=TRUE</code> . The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
<code>summary_list</code>	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
<code>PCC_PLOT</code>	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
<code>PriorPosterior_PLOT</code>	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
<code>historical_data_PLOT</code>	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
<code>pdf_report</code>	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
<code>path_pdf_report</code>	specified path to save pdf report if <code>pdf_report=TRUE</code> . The default path is the temporary direction <code>tempdir()</code> .
<code>xlab, ylab, main</code>	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC LogNormal with unknown variance" respectively

## Details

Inorm2\_PCC provides the Predictive Control Chart (PCC) process for LogNormal data, when the shape parameter is unknown and the scale parameter is known. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPd), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown shape parameter is an Inverse Gamma:  $IG(a_0, b_0)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $IG(0, 0)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either  $ARL_0$  or FAP.

## Value

Runs the PCC process, while if  $FIR=T$ , then the FIR-PCC is implemented. If  $plot=T$ , then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPd/M) region, and an indication in case of alarms. If  $PCC\_PLOT=T$ , then the PCC process is visualized, while if  $historical\_data\_PLOT=T$ , the historical data are added to the plot. If  $PriorPosterior\_PLOT = T$ , a plot with the prior and the posterior is provided. Finally, if  $pdf\_report=T$ , all the output is reported and saved into a pdf file.

## Examples

```
# 30 LogNormal observations introducing an outlier at the 15th observation
set.seed(1)
SimData <- rlnorm(n = 30, meanlog = 0, sdlog = 1/2)
SimData[15] <- SimData[15] + 3*sqrt( exp(1/4)*( exp(1/4)-1 ) )
plot(SimData)
Inorm2_PCC(SimData, ml = 0)
```

---

Inorm3\_PCC

*PCC for LogNormal data with both parameters unknown*

---

## Description

Inorm3\_PCC is used to derive the Predictive Control Chart (PCC) for LogNormal data, when both the scale and the shape parameters are unknown.

## Usage

```
Inorm3_PCC(data = NULL, historical_data = NULL,
            mu0 = 0, l0 = 0, a0 = -1/2, b0 = 0, alpha_0 = NULL,
            ARL_0 = 370.4, FAP = NULL, FIR = FALSE, ffir = .99, afir = 1/8,
            summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
            historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
```

```
xlab = "Observation Order", ylab = "Quality characteristic Values",
main = "PCC LogNormal with unknown parameters")
```

### Arguments

<code>data</code>	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
<code>historical_data</code>	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
<code>mu0</code>	scalar; hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
<code>l0</code>	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
<code>a0</code>	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is -1/2 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
<code>b0</code>	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
<code>alpha_0</code>	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.
<code>ARL_0</code>	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
<code>FAP</code>	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of ARL_0. It is based on the Sidak's correction.
<code>FIR</code>	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
<code>ffFIR</code>	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
<code>aFIR</code>	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
<code>summary_list</code>	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
<code>PCC_PLOT</code>	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
<code>PriorPosterior_PLOT</code>	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.



historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC LogNormal with unknown parameters" respectively.

### Details

Inorm3\_PCC provides the Predictive Control Chart (PCC) process for LogNormal data, when both the scale and the shape parameters are unknown. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The joint initial prior for the unknown parameters is a Normal Inverse Gamma:  $NIG(\mu_0, l_0, a_0, b_0)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $NIG(0, 0, -1/2, 0)$ , without the use of historical data. In this special case, the PCC testing starts at the third observation.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either  $ARL_0$  or FAP.

### Value

Runs the PCC process, while if  $FIR=T$ , then the FIR-PCC is implemented. If  $plot=T$ , then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, and an indication in case of alarms. If  $PCC\_PLOT=T$ , then the PCC process is visualized, while if  $historical\_data\_PLOT=T$ , the historical data are added to the plot. If  $Prior\_Posterior\_PLOT = T$ , a plot with the prior and the posterior is provided. Finally, if  $pdf\_report=T$ , all the output is reported and saved into a pdf file.

### Examples

```
# 30 LogNormal observations introducing an outlier at the 15th observation
ssddl1 <- 1/2
set.seed(9)
SimData <- rlnorm( n = 30, meanlog = 0, sdlog = ssddl1)
SimData[15] <- SimData[15] + 3*sqrt( exp(ssddl1^2)*( exp(ssddl1^2)-1 ) )
plot(SimData)
Inorm3_PCC(SimData)
```

---

lnorm\_HD

*The Highest Density (HD) interval of Lognormal distribution.*


---

### Description

lnorm\_HD is used to derive the narrowest interval of a predetermined coverage from a Lognormal distribution, containing the Highest Density (HD) values.

### Usage

```
lnorm_HD( cover = NULL, mulog = NULL, sdlog = NULL, plot = FALSE,
          xlab = "x", ylab = "Density" )
```

### Arguments

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Density (HD).
mulog	scalar (real); It is the scale parameter of the Lognormal distribution and needs to be a number.
sdlog	scalar (positive); It is the shape parameter of the Lognormal distribution and needs to be a number.
plot	logical; if TRUE, the plot of the HD region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HD plot. The default values are "x" and "Density".

### Details

lnorm\_HD provides the Highest Density (HD) of a Lognormal distribution. HD is the narrowest interval with a predetermined coverage. The values included in HD have density larger than every point outside the region.

### Value

Returns a data frame with the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

### Examples

```
lnorm_HD(0.95, 0, 1/2, plot = TRUE)
```

---

lt_HD	<i>The Highest Density (HD) interval of Logt distribution.</i>
-------	--

---

### Description

lt\_HD is used to derive the narrowest interval of a predetermined coverage from a Logt distribution, containing the Highest Density (HD) values.

### Usage

```
lt_HD( cover = NULL, df = NULL, mulog = NULL, sdlog = NULL, plot = FALSE,
       xlab = "x", ylab = "Density" )
```

### Arguments

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Density (HD). The default value is 0.95.
df	scalar (positive); It is the degrees of freedom of the Logt distribution and needs to be a number.
mulog	scalar (real); It is the scale parameter of the Logt distribution and needs to be a number. The default value is 0.
sdlog	scalar (positive); It is the shape parameter of the Logt distribution and needs to be a number. The default value is 1.
plot	logical; if TRUE, the plot of the HD region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HD plot. The default values are "x" and "Density".

### Details

lt\_HD provides the Highest Density (HD) of a Logt distribution. HD is the narrowest interval with a pretermined coverage. The values included in HD have density larger than every point outside the region.

### Value

Returns a data frame with the upper and the lower limits of the Highest Predictive Denisty or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

### Examples

```
lt_HD(0.95, 10, 0, 1/2, plot = TRUE)
```

---

nbinom_HM	<i>The Highest Mass (HM) interval of Beta-Negative Binomial distribution.</i>
-----------	---

---

### Description

nbinom\_HM is used to derive the narrowest interval of a predetermined coverage from a Negative Binomial distribution, containing the Highest Mass (HM) values.

### Usage

```
nbinom_HM( cover = NULL, r = NULL, p = NULL, plot = FALSE,
           xlab = "x", ylab = "Probability" )
```

### Arguments

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Mass (HM).
r	scalar (integer); It is the number of failures until the experiment is stopped.
p	scalar (between 0 and 1); It is the probability of success for each Bernoulli trial.
plot	logical; if TRUE, the plot of the HM region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HM plot. The default values are "x" and "Probability".

### Details

nbinom\_HM provides the Highest Mass (HM) of a Negative Binomial distribution. HM is the narrowest region, with the minimum absolute difference from a pretermined coverage. The values included in HM have probability larger than every point outside the region.

### Value

Returns a data frame with the upper and the lower limits of the Highest Predictive Denisty or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

### Examples

```
nbinom_HM(0.95, 4, 0.2, plot = TRUE)
```

nbinom\_PCC

*PCC for Negative Binomial data with probability parameter unknown***Description**

nbinom\_PCC is used to derive the Predictive Control Chart (PCC) for Negative Binomial data, when the probability parameter is unknown and the number of failures is known.

**Usage**

```
nbinom_PCC( data = NULL, historical_data = NULL,
            r1 = NULL, a0 = 0, b0 = 1/2, alpha_0 = NULL,
            ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR = 1/8,
            summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
            historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
            xlab = "Observations", ylab = "Quality characteristic Values",
            main = "PCC Negative Binomial with unknown probability" )
```

**Arguments**

data	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
r1	scalar (positive); parameter of the likelihood. It is the known number of failures of the data and needs to be a number.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
a0	scalar (positive); scalar (positive); shape hyperparameter of the prior Beta(a, b). It needs to be a number. The default is 0 and it refers to the initial reference prior Beta(0, 1/2).
b0	scalar (positive); scalar (positive); shape hyperparameter of the prior Beta(a, b). It needs to be a number. The default is 1/2 and it refers to the initial reference prior Beta(0, 1/2).
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is 1/n_0, where n_0 is the size of the historical data.
ARL_0	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
FAP	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of ARL_0. It is based on the Sidak's correction.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.

ffir	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
aFIR	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC Negative Binomial with unknown probability" respectively

## Details

nbinom\_PCC provides the Predictive Control Chart (PCC) process for Negative Binomial data, when the probability parameter is unknown and the number of failures is known. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPd), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown probability is a Beta:  $\text{Beta}(a_0, b_0)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $\text{Beta}(0, 1/2)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either  $\text{ARL}_0$  or FAP.

## Value

Runs the PCC process, while if FIR=T, then the FIR-PCC is implemented. If plot=T, then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPd/M) region, and an indication in case of alarms. If PCC\_PLOT=T, then the PCC process

is visualized, while if `historical_data_PLOT=T`, the historical data are added to the plot. If `Prior-Posterior_PLOT = T`, a plot with the prior and the posterior is provided. Finally, if `pdf_report=T`, all the output is reported and saved into a pdf file.

### Examples

```
# 30 Negative Binomial observations introducing an outlier at the 15th observation
set.seed(5)
SimData <- rlnbinom(n = 30, size = 10, prob = 0.9)
SimData[15] <- round( SimData[15] + 3*sqrt(10*0.1/(0.9^2)) )
nbinom_PCC(SimData, r1 = 10)
```

---

norm1\_PCC

*PCC for Normal data with mean unknown*

---

### Description

`norm1_PCC` is used to derive the Predictive Control Chart (PCC) for Normal data, when the mean is unknown and the variance is known.

### Usage

```
norm1_PCC(data = NULL, historical_data = NULL, sd1 = NULL,
          mu0 = 0, sd0 = 10^6, alpha_0 = NULL,
          ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR=1/8,
          summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
          historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
          xlab = "Observations", ylab = "Quality characteristic Values",
          main="PCC Normal with unknown mean")
```

### Arguments

<code>data</code>	vector; a dataset for PCC implementation. Data needs to be in a vector form.
<code>historical_data</code>	vector; an optional dataset of historical data. Historical data needs to be in a vector form.
<code>sd1</code>	scalar (positive); parameter of the likelihood. It is the known standard deviation of the data and needs to be a number.
<code>mu0</code>	scalar; hyperparameter of the prior $N(\mu_0, sd_0^2)$ . It needs to be a number. The default is 0 and it refers to the initial reference prior $N(0, \infty)$ .
<code>sd0</code>	scalar (positive); hyperparameter of the prior $N(\mu_0, sd_0^2)$ . It needs to be a number. The default is $10^6$ and it refers to the initial reference prior $N(0, \infty)$ .
<code>alpha_0</code>	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.

ARL_0	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
FAP	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of ARL_0. It is based on the Sidak's correction.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
fFIR	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
aFIR	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC Normal with unknown mean" respectively.

## Details

norm1\_PCC provides the Predictive Control Chart (PCC) process for Normal data, when the mean is unknown and the variance is known. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown mean is a Normal:  $N(\mu_0, \sigma_0^2)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $N(0, \infty)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either ARL\_0 or FAP.



**Value**

Runs the PCC process, while if FIR=T, then the FIR-PCC is implemented. If plot=T, then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, and an indication in case of alarms. If PCC\_PLOT=T, then the PCC process is visualized, while if historical\_data\_PLOT=T, the historical data are added to the plot. If Prior-Posterior\_PLOT = T, a plot with the prior and the posterior is provided. Finally, if pdf\_report=T, all the output is reported and saved into a pdf file.

**Examples**

```
# 30 Normal observations introducing an outlier of 3*sd at the 15th observation
set.seed(1234)
out <- rnorm(30)
out[15] <- out[15] + 3
norm1_PCC(out, sd1 = 1)

# Real data application
attach(aPTT)
norm1_PCC(data = aPTT_current, historical_data = aPTT_historical, sd1 = 0.57)
```

---

norm2\_PCC

*PCC for Normal data with variance unknown*


---

**Description**

norm2\_PCC is used to derive the Predictive Control Chart (PCC) for Normal data, when the variance is unknown and the mean is known.

**Usage**

```
norm2_PCC(data = NULL, historical_data = NULL, m1 = NULL,
          a0 = 0, b0 = 0, alpha_0 = NULL,
          ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR = 1/8,
          summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
          historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
          xlab = "Observation Order", ylab = "Quality characteristic Values",
          main = "PCC Normal with unknown variance")
```

**Arguments**

**data** vector; a dataset for PCC implementation. Data needs to be in a vector form.

**historical\_data** vector; an optional dataset of historical data. Historical data needs to be in a vector form.

**m1** scalar; parameter of the likelihood. It is the known mean of the data and needs to be a number.

a0	scalar (positive); hyperparameter of the prior IG(a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior IG(0, 0).
b0	scalar (positive); hyperparameter of the prior IG(a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior IG(0, 0).
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is 1/n_0, where n_0 is the size of the historical data.
ARL_0	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
FAP	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of ARL_0. It is based on the Sidak's correction.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
fFIR	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
aFIR	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted precending the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC Normal with unknown variance" respectively

## Details

norm2\_PCC provides the Predictive Control Chart (PCC) process for Normal data, when the variance is unknown and the mean is known. The PCC process is based on the sequential testing of the

future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown variance is an Inverse Gamma:  $IG(a_0, b_0)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $IG(0, 0)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either  $ARL_0$  or  $FAP$ .

### Value

Runs the PCC process, while if  $FIR=T$ , then the FIR-PCC is implemented. If  $plot=T$ , then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, and an indication in case of alarms. If  $PCC\_PLOT=T$ , then the PCC process is visualized, while if  $historical\_data\_PLOT=T$ , the historical data are added to the plot. If  $PriorPosterior\_PLOT = T$ , a plot with the prior and the posterior is provided. Finally, if  $pdf\_report=T$ , all the output is reported and saved into a pdf file.

### Examples

```
# 30 Normal observations introducing an outlier of 3*sd at the 15th observation
set.seed(1234)
out <- rnorm(30)
out[15] <- out[15] + 3
norm2_PCC(out, ml = 0)

# Real data application
attach(aPTT)
norm2_PCC(data = aPTT_current, historical_data = aPTT_historical, ml = 30)
```

---

norm3\_PCC

*PCC for Normal data with both parameters unknown*

---

### Description

norm3\_PCC is used to derive the Predictive Control Chart (PCC) for Normal data, when both the mean and the variance are unknown.

### Usage

```
norm3_PCC(data = NULL, historical_data = NULL,
          mu0 = 0, l0 = 0, a0 = -1/2, b0 = 0, alpha_0 = NULL,
          ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR = 1/8,
          summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
          historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
          xlab = "Observation Order", ylab = "Quality characteristic Values",
          main = "PCC Normal with unknown parameters")
```

**Arguments**

data	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
mu0	scalar; hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
l0	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
a0	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is -1/2 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
b0	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is 1/n_0, where n_0 is the size of the historical data.
ARL_0	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
FAP	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of ARL_0. It is based on the Sidak's correction.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
fFIR	a number between 0 and 1; It is used if FIR=TRUE. The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
aFIR	non-negative number; It is used if FIR=TRUE. The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.

pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC Normal with unknown parameters" respectively.

## Details

norm3\_PCC provides the Predictive Control Chart (PCC) process for Normal data, when both the mean and the variance are unknown. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The joint initial prior for the unknown parameters is a Normal Inverse Gamma:  $NIG(\mu_0, l_0, a_0, b_0)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $NIG(0, 0, -1/2, 0)$ , without the use of historical data. In this special case, the PCC testing starts at the third observation.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either  $ARL_0$  or  $FAP$ .

## Value

Runs the PCC process, while if  $FIR=T$ , then the FIR-PCC is implemented. If  $plot=T$ , then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, and an indication in case of alarms. If  $PCC\_PLOT=T$ , then the PCC process is visualized, while if  $historical\_data\_PLOT=T$ , the historical data are added to the plot. If  $Prior-Posterior\_PLOT = T$ , a plot with the prior and the posterior is provided. Finally, if  $pdf\_report=T$ , all the output is reported and saved into a pdf file.

## Examples

```
# 30 Normal observations introducing an outlier of 3*sd at the 15th observation
set.seed(1234)
out <- rnorm(30)
out[15] <- out[15] + 3
norm3_PCC(out)

# Real data application
attach(aPTT)
norm3_PCC(data = aPTT_current, historical_data = aPTT_historical)
```

---

 norm\_HD

*The Highest Density (HD) interval of Normal distribution.*


---

### Description

norm\_HD is used to derive the narrowest interval of a predetermined coverage from a Normal distribution, containing the Highest Density (HD) values.

### Usage

```
norm_HD( cover = NULL, mu = NULL, sdv = NULL, plot = FALSE,
         xlab = "x", ylab = "Density" )
```

### Arguments

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Density (HD).
mu	scalar (real); It is the mean of the Normal distribution and needs to be a number.
sdv	scalar (positive); It is the standard deviation of the Normal distribution and needs to be a number.
plot	logical; if TRUE, the plot of the HD region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HD plot. The default values are "x" and "Density".

### Details

norm\_HD provides the Highest Density (HD) of a Normal distribution. HD is the narrowest interval with a predetermined coverage. The values included in HD have density larger than every point outside the region.

### Value

Returns a data frame with the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

### Examples

```
norm_HD(0.95, mu = 10, sdv = 1/2, plot = TRUE)
```

---

norm_mean2_PRC	<i>PRC for Normal data with unknown parameters (mean)</i>
----------------	---

---

### Description

norm\_mean2\_PRC is used to derive the Predictive Ratio CUSUM (PRC) for the mean of Normal data, when both the mean and the variance are unknown.

### Usage

```
norm_mean2_PRC( data = NULL, historical_data = NULL, mu0 = 0, l0 = 0,
  a0 = -1/2, b0 = 0, alpha_0 = NULL, k = 1, two.sided = FALSE,
  h = log(100), FIR = FALSE, ffir = 1/2, dfir = 3/4,
  summary_list = TRUE, PRC_PLOT = TRUE, pdf_report = FALSE,
  path_pdf_report = tempdir(), xlab = "Observation Order",
  ylab = "PRC cumulative statistics",
  main = "PRC Normal with unknown parameters (mean model)" )
```

### Arguments

data	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
mu0	scalar; hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
l0	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
a0	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is -1/2 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
b0	scalar (positive); hyperparameter of the prior NIG(mu0, l0, a0, b0). It needs to be a number. The default is 0 and it refers to the initial reference prior NIG(0, 0, -1/2, 0).
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is 1/n_0, where n_0 is the size of the historical data.
k	scalar (positive); tuning parameter, which represents deviation between IC and OOC state. Precisely, if k>1 then the OOC state corresponds to a k standard deviations increase in the mean and if k<1 corresponds to a k standard deviations decrease. The default value is 1.

two.sided	logical; If TRUE, then a two sided PRC is employed with tuning parameters $k$ and $1/k$ respectively. It is FALSE by default.
h	scalar (positive); Decision limit and we raise an alarm if the cumulative statistic exceeds it (or $2h$ for two sided shifts). The default value is $\log(100)=4.605$ , for which we have a decisive cumulative evidence in favor of the OOC state.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PRC is applied, which is an exponentially decreasing adjustment for the initial first tests, inflating the PRC statistics.
fFIR	non-negative number; It is used if FIR=TRUE. It represents the proportion of the inflation for the PRC statistic in the for the first test. The default value is $1/2$ .
dFIR	a number between 0 and 1; It is used if FIR=TRUE. It is a smoothing parameter, specifying the exponential decay of the adjustment (the smaller the $d$ the fastest the decay). The default value is $3/4$ .
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PRC_PLOT	logical; if TRUE, the PRC plot is displayed. It is TRUE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "PRC cumulative statistics" and "PRC Binomial with unknown probability" respectively

## Details

norm\_mean2\_PRC provides the Predictive Ratio CUSUM (PRC) process for the mean of Normal data, when both the mean and the variance are unknown. The PRC process is based on sequential comparison between the IC and OOC state, via their predictive distributions. The parameter  $k$  represents the deviance between these two states. The default value is  $k=1$ , which corresponds to one standard deviation increase in the mean. The testing starts as early as the second observation, apart from when using the default non-informative prior, without historical data. In this special case, the PCC testing starts at the third observation.

The initial prior for the unknown probability is a Normal Inverse Gamma:  $NIG(\mu_0, 10, a_0, b_0)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $NIG(0, 0, -1/2, 0)$ , without the use of historical data.

A FIR option is available by inflating the first few PRC statistics. The choice of the decision limit  $h$  reflects the false alarm tolerance.



**Examples**

```

# the PRC process for the first application in
# "Design and properties of the Predictive Ratio Cusum (PRC) control charts"

### CD: Current data (New reagent)
### HD: Historical data (Previous reagent)

CD <- c( 31.0, 30.0, 32.0, 28.0, 33.2, 33.2, 35.1, 35.1, 33.9, 37.9,
         33.2, 36.5, 33.2, 35.1, 34.5, 36.5, 33.2, 35.1, 37.2, 32.6, 36.5 )
HD <- c( 31, 30, 33, 30, 33, 30, 31, 32, 32, 30, 33, 31, 34, 31, 34, 34, 36, 30,
         33, 29, 34, 32, 32, 28, 34, 32, 32, 30, 31, 29, 31, 29, 31, 32,34,34,32 )

N <- length(CD)
n0 <- length(HD)

### initial prior parameters

M0F <- 31.8
LF <- 1/2
AF <- 2
BF <- 2.1^2

norm_mean2_PRC( data = CD, historical_data = HD, alpha_0 = 1/n0, mu0 = M0F,
                l0 = LF, a0 = AF, b0 = BF, h = 3.749, two.sided = TRUE )

### a real data application to aPTT values

### CURRENT DATA aPTT
CD <- c( 29.0, 29.1, 28.7, 28.2, 28.0, 29.1, 28.6, 28.7, 28.6, 29.0, 28.4,
         28.1, 28.8, 29.7, 28.8, 29.8, 28.8, 29.4, 28.4, 28.7, 28.7, 29.5,
         28.5, 28.4, 28.1, 28.6, 28.2, 29.6, 28.9, 29.1, 29.0, 29.9, 28.6,
         29.3, 28.2, 28.6, 27.6, 27.3, 28.7, 27.2, 28.4, 28.0, 28.4, 27.8,
         28.4, 28.4, 27.7, 29.2, 27.5, 27.7)

### HISTORICAL DATA aPTT
HD <- c( 28.0, 28.9, 27.7, 29.3, 28.9, 29.5, 28.2, 27.5, 28.8, 28.9, 28.7,
         27.4, 28.6, 28.5, 29.6, 28.7, 21.3, 29.4, 28.1, 28.9, 28.3, 27.6,
         29.0, 29.2, 27.8, 29.1, 28.9, 29.4, 29.4, 28.9, 28.9, 29.2, 29.4,
         29.4, 28.1, 28.5, 29.7, 29.3, 28.6, 29.2, 29.3, 29.3, 29.3, 30.0,
         29.1, 29.1, 26.8, 29.0, 29.3, 28.3)

norm_mean2_PRC( data = CD, historical_data = HD, mu0 = 28.9,
                l0 = 1/4, a0 = 2, b0 = 0.49, two.sided = TRUE )

```

---

norm_mean2_PRC_h	<i>Derivation of the decision limit for the PRC for Normal data with unknown parameters (mean)</i>
------------------	--

---

### Description

norm\_mean2\_PRC\_h is used to estimate the decision limit  $h$  for the Predictive Ratio CUSUM (PRC) for the mean of Normal data, when both the mean and the variance are unknown. The process controls the false alarms tolerance, providing the ARL<sub>0</sub> or the FAP, based on the standardized predictive distribution.

### Usage

```
norm_mean2_PRC_h( ARL_0 = 370.4, FAP = NULL, N=NULL, historical_data = NULL,
  l0 = 0, a0 = -1/2, alpha_0 = NULL, k = 1, it = 1e4,
  ARL0tol = 10/it )
```

### Arguments

ARL_0	scalar (positive); lower bound of the In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4.
FAP	scalar (between 0 and 1); lower bound of the False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of $N$ hypothesis tests and it can be used instead of ARL <sub>0</sub> .
N	scalar (natural); number of $N$ hypothesis tests. It is used for the FAP determination.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
l0	scalar (positive); shape hyperparameter of the prior Beta( $a$ , $b$ ). It needs to be a number. The default is 0.
a0	scalar (positive); shape hyperparameter of the prior Beta( $a$ , $b$ ). It needs to be a number. The default is -1/2.
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.
k	scalar (positive); tuning parameter, which represents deviation between IC and OOC state. Precisely, if $k > 1$ then the OOC state corresponds to a $k$ standard deviations increase in the mean and if $k < 1$ corresponds to a $k$ standard deviations decrease. The default value is 1.
it	scalar (natural); iterations of the simulation. The default value is 10000.
ARL0tol	scalar (positive); relative tolerance of ARL <sub>0</sub> estimate. It represents the maximum amount of error which is allowed. The default value is 10/it.

## Details

norm\_mean2\_PRC\_h estimates the decision limit  $h$  regarding the desired upper bound of false alarms tolerance for the Predictive Ratio CUSUM (PRC) process for the mean of Normal data, when both the mean and variance are unknown. The estimate concerns an one-sided PRC process and the default setup corresponds to one standard deviation increase in the mean. The estimate is based on the use of the standardized predictive distribution and the incorporation of historical data is possible via the power prior.

The default metric of the false alarms tolerance is  $ARL_0=370.4$ , while a FAP option is available. The decision limit is obtained via the numerical method regula falsi when  $ARL_0$  is selected, while simulated samples of fixed size are used when FAP is of interest.

## Examples

```
# Derivation of the decision limit of the first application in
# "Design and properties of the Predictive Ratio Cusum (PRC) control charts"

CD <- c( 31.0, 30.0, 32.0, 28.0, 33.2, 33.2, 35.1, 35.1, 33.9, 37.9,
        33.2, 36.5, 33.2, 35.1, 34.5, 36.5, 33.2, 35.1, 37.2, 32.6, 36.5 )
HD <- c( 31, 30, 33, 30, 33, 30, 31, 32, 32, 30, 33, 31, 34, 31, 34, 34, 36, 30,
        33, 29, 34, 32, 32, 28, 34, 32, 32, 30, 31, 29, 31, 29, 31, 32, 34, 34, 32 )

N <- length(CD)
n0 <- length(HD)
Pa0 <- 1/n0
M0F <- 31.8
LF <- 1/2
AF <- 2
BF <- 2.1^2
M0F ; LF ; AF ; BF

# To replicate results from application set 'it = 1e5'
norm_mean2_PRC_h( ARL_0 = NULL, FAP = 0.05, N = N, l0 = LF, a0 = AF,
                  historical_data = HD, alpha_0 = Pa0, it = 1e4 )
```

---

pois\_PCC

*PCC for Poisson data with rate parameter unknown*

---

## Description

pois\_PCC is used to derive the Predictive Control Chart (PCC) for Poisson data, when the rate parameter is unknown.

**Usage**

```
pois_PCC( data = NULL, s = NULL, historical_data = NULL,
          historical_s = NULL, c0 = 1/2, d0 = 0, alpha_0 = NULL,
          ARL_0 = 370.4, FAP = NULL, FIR = FALSE, fFIR = .99, aFIR = 1/8,
          summary_list = TRUE, PCC_PLOT = TRUE, PriorPosterior_PLOT = FALSE,
          historical_data_PLOT = FALSE, pdf_report = FALSE, path_pdf_report = tempdir(),
          xlab = "Observation Order", ylab = "Quality characteristic Values",
          main = "PCC Poisson with rate unknown" )
```

**Arguments**

<code>data</code>	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
<code>s</code>	vector; refers to the period of time or grouping of counts, if data are rate dataset. It needs to be in a vector form.
<code>historical_data</code>	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
<code>historical_s</code>	vector; refers to the period of time or grouping of counts, if <code>historical_data</code> are rate dataset. It needs to be in a vector form.
<code>c0</code>	scalar (positive); scalar (positive); hyperparameter of the prior $G(c_0, d_0)$ . It needs to be a number. The default is 1/2 and it refers to the initial reference prior $G(1/2, 0)$ .
<code>d0</code>	scalar (positive); scalar (positive); hyperparameter of the prior $G(c_0, d_0)$ . It needs to be a number. The default is 0 and it refers to the initial reference prior $G(1/2, 0)$ .
<code>alpha_0</code>	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.
<code>ARL_0</code>	scalar (positive); In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4
<code>FAP</code>	scalar (between 0 and 1); False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of $N$ hypothesis tests and it can be used instead of <code>ARL_0</code> . It is based on the Sidak's correction.
<code>FIR</code>	logical; If TRUE, then the Fast Initial Response (FIR) PCC is applied, which is an adjustment (narrows the PCC limits) for the initial first tests.
<code>fFIR</code>	a number between 0 and 1; It is used if <code>FIR=TRUE</code> . The default value is 0.99 and represents the proportion of the adjusted PCC region over the initial one for the first test.
<code>aFIR</code>	non-negative number; It is used if <code>FIR=TRUE</code> . The default value is 0.125 and it is a smoothing parameter for the FIR adjustment.
<code>summary_list</code>	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.

PCC_PLOT	logical; if TRUE, the PCC plot is displayed. It is TRUE by default.
PriorPosterior_PLOT	logical; if TRUE, the Prior and Posterior distribution(s) of the parameter(s) are displayed. It is TRUE by default.
historical_data_PLOT	logical; if TRUE, the historical data are plotted preceding the PCC plot. It is FALSE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if pdf_report=TRUE. The default path is the temporary direction tempdir().
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "Quality characteristic Values" and "PCC Poisson" respectively

## Details

pois\_PCC provides the Predictive Control Chart (PCC) process for Poisson data, when the rate parameter is unknown. The PCC process is based on the sequential testing of the future observable against the Highest Predictive Density (HPrD), which is obtained by the posterior predictive distribution. The PCC testing starts as early as the second observation.

The initial prior for the unknown rate is a Gamma:  $G(c, d)$ , where  $d$  is the rate parameter. Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $G(1/2, 0)$ , without the use of historical data.

A FIR option is available by narrowing the first few control limits. The metrics that can be used to control the false alarms tolerance are either  $ARL_0$  or FAP.

## Value

Runs the PCC process, while if  $FIR=T$ , then the FIR-PCC is implemented. If  $plot=T$ , then a data frame is provided with the data, the upper and the lower limits of the Highest Predictive Density or Mass (HPrD/M) region, and an indication in case of alarms. If  $PCC\_PLOT=T$ , then the PCC process is visualized, while if  $historical\_data\_PLOT=T$ , the historical data are added to the plot. If  $PriorPosterior\_PLOT = T$ , a plot with the prior and the posterior is provided. Finally, if  $pdf\_report=T$ , all the output is reported and saved into a pdf file.

## Examples

```
# 30 Poisson observations introducing an outlier at the 15th observation
set.seed(1111)
out <- rpois(n = 30, lambda = 4)
out[15] <- out[15] + 6
pois_PCC(out)
```

```
# Real data application
attach(ECE)
pois_PCC(data = defect_counts, s = inspected_units)
```

---

pois\_PRC

*PRC for Poisson data with rate parameter unknown*


---

### Description

pois\_PRC is used to derive the Predictive Ratio CUSUM (PRC) for Poisson data, when the rate parameter is unknown.

### Usage

```
pois_PRC( data = NULL, s = NULL, historical_data = NULL, historical_s = NULL,
  c0 = 1/2, d0 = 0, alpha_0 = NULL, k=2, two.sided=FALSE,
  h = log(100), FIR = FALSE, fFIR = 1/2, dFIR = 3/4,
  summary_list = TRUE, PRC_PLOT = TRUE, pdf_report = FALSE,
  path_pdf_report = tempdir(),
  xlab = "Observation Order", ylab = "PRC cumulative statistics",
  main = "PRC Poisson with unknown rate" )
```

### Arguments

data	vector; a univariate dataset for PCC implementation. Data needs to be in a vector form.
s	vector; refers to the period of time or grouping of counts, if data are rate dataset. It needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.
historical_s	vector; refers to the period of time or grouping of counts, if historical_data are rate dataset. It needs to be in a vector form.
c0	scalar (positive); scalar (positive); hyperparameter of the prior $G(c_0, d_0)$ . It needs to be a number. The default is 1/2 and it refers to the initial reference prior $G(1/2, 0)$ .
d0	scalar (positive); scalar (positive); hyperparameter of the prior $G(c_0, d_0)$ . It needs to be a number. The default is 0 and it refers to the initial reference prior $G(1/2, 0)$ .
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is $1/n_0$ , where $n_0$ is the size of the historical data.
k	scalar (positive); tuning parameter, which represents deviation between IC and OOC state. Precisely, if $k > 1$ then the OOC state corresponds to a $(k-1)100\%$ increase in the rate parameter and if $k < 1$ corresponds $(1-k)100\%$ decrease. The default value is 2.

two.sided	logical; If TRUE, then a two sided PRC is employed with tuning parameters $k$ and $1/k$ respectively. It is FALSE by default.
h	scalar (positive); Decision limit and we raise an alarm if the cumulative statistic exceeds it (or $2h$ for two sided shifts). The default value is $\log(100)=4.605$ , for which we have a decisive cumulative evidence in favor of the OOC state.
FIR	logical; If TRUE, then the Fast Initial Response (FIR) PRC is applied, which is an exponentially decreasing adjustment for the initial first tests, inflating the PRC statistics.
fFIR	non-negative number; It is used if $FIR=TRUE$ . It represents the proportion of the inflation for the PRC statistic in the for the first test. The default value is $1/2$ .
dFIR	a number between 0 and 1; It is used if $FIR=TRUE$ . It is a smoothing parameter, specifying the exponential decay of the adjustment (the smaller the $d$ the fastest the decay). The default value is $3/4$ .
summary_list	logical; If it is TRUE, then a data frame is provided, containing the data sequence, the PCC limits along with the possible occurrence of an alarm. It is TRUE by default.
PRC_PLOT	logical; if TRUE, the PRC plot is displayed. It is TRUE by default.
pdf_report	logical; if TRUE then the summary list, PCC plot and Prior/Posterior distribution(s) of the parameter(s) are reported in a pdf file.
path_pdf_report	specified path to save pdf report if $pdf\_report=TRUE$ . The default path is the temporary direction <code>tempdir()</code> .
xlab, ylab, main	The titles of the x-axis, y-axis and the overall title for the PCC plot. The default values are "Observation Order", "PRC cumulative statistics" and "PRC Binomial with unknown probability" respectively

## Details

pois\_PRC provides the Predictive Ratio CUSUM (PRC) process for Poisson data, when the rate parameter parameter is unknown. The PRC process is based on sequential comparison between the IC and OOC state, via their predictive distributions. The parameter  $k$  represents the deviance between these two states. The default value is  $k=2$ , which corresponds to a doubling of the rate parameter. The testing starts as early as the second observation.

The initial prior for the unknown probability is a Gamma:  $G(c_0, d_0)$ . Furthermore, the direct use of available historical data is possible via the power prior, if they are available. In this case, the default value for the power prior parameter  $\alpha_0$  is the reciprocal of the length of the historical data, which conveys the weight of a single observation to the prior information. The default prior is the non-informative reference prior  $G(1/2, 0)$ , without the use of historical data.

A FIR option is available by inflating the first few PRC statistics. The choice of the decision limit  $h$  reflects the false alarm tolerance.

## Examples

```
# the PRC process for the second application in
# "Design and properties of the Predictive Ratio Cusum (PRC) control charts"
```

```

### CURRENT DATA
CD <- c(1, 0, 0, 0, 1, 0, 3, 3, 3, 2, 5, 5, 2, 4, 4, 3, 4, 3, 8, 3, 2, 2)

### product exposures per million

sn <- c( 0.206, 0.313, 0.368, 0.678, 0.974, 0.927, 0.814, 0.696, 0.659, 0.775, 0.731,
         0.710, 0.705, 0.754, 0.682, 0.686, 0.763, 0.833, 0.738, 0.741, 0.843, 0.792 )

# regular process
pois_PRC(data = CD, s = sn)

# FIR process
pois_PRC(data = CD, s = sn, FIR = TRUE)

```

---

pois\_PRC\_h

*Derivation of the decision limit for the PRC for Poisson data with probability parameter unknown*

---

## Description

pois\_PRC\_h is used to estimate the decision limit h for the Predictive Ratio CUSUM (PRC) for Poisson data. The process controls the false alarms tolerance, providing a lower bound of ARL<sub>0</sub> or FAP, based on the marginal distribution.

## Usage

```

pois_PRC_h( ARL_0 = 370.4, FAP = NULL, N = NULL, s = NULL,
            historical_data = NULL, historical_s = NULL,
            c0 = NULL, d0 = NULL, alpha_0 = NULL, k = 2,
            it = 1e4, ARL0tol = 10/it )

```

## Arguments

ARL <sub>0</sub>	scalar (positive); lower bound of the In Control (IC) Average Run Length (ARL). It is average number of IC data points that we will plot in the PCC before a false alarm occurs. The default value is 370.4.
FAP	scalar (between 0 and 1); lower bound of the False Alarm Probability (FAP). It is the probability of raising at least one false alarm out of a pre-determined number of N hypothesis tests and it can be used instead of ARL <sub>0</sub> .
N	scalar (natural); number of N hypothesis tests. It is used for the FAP determination.
s	vector; refers to the period of time or grouping of counts, if data are rate dataset. It needs to be in a vector form.
historical_data	vector; an optional univariate dataset of historical data. Historical data needs to be in a vector form.



historical_s	vector; refers to the period of time or grouping of counts, if historical_data are rate dataset. It needs to be in a vector form.
c0	scalar (positive); scalar (positive); hyperparameter of the prior G(c0, d0). It needs to be a number.
d0	scalar (positive); scalar (positive); hyperparameter of the prior G(c0, d0). It needs to be a number.
alpha_0	scalar (non negative); It is a power prior parameter controlling the influence of the historical data on the posterior distribution. The default is 1/n_0, where n_0 is the size of the historical data.
k	scalar (positive); tuning parameter, which represents deviation between IC and OOC state. Precisely, if k>1 then the OOC state corresponds to a (k-1)100% increase in the rate parameter and if k<1 corresponds (1-k)100% decrease. The default value is 2.
it	scalar (natural); iterations of the simulation. The default value is 10000.
ARL0tol	scalar (positive); relative tolerance of ARL_0 estimate. It represents the maximum amount of error which is allowed. The default value is 10/it.

### Details

pois\_PRC\_h estimates the decision limit h regarding the desired upper bound of false alarms tolerance for the Predictive Ratio CUSUM (PRC) process for Binomial data, when the probability parameter is unknown. The estimate concerns an one-sided PRC process and the default setup corresponds to a doubling of the rate parameter (k=2). The estimate is based on the use of the marginal distribution (prior predictive) and the incorporation of historical data is possible via the power prior. The default metric of the false alarms tolerance is ARL\_0=370.4, while a FAP option is available. The decision limit is obtained via the numerical method regula falsi when ARL\_0 is selected, while simulated samples of fixed size are used when FAP is of interest.

### Examples

```
pois_PRC_h(ARL_0 = 150, c0 = 40, d0 = 10, it = 1e3)
```

---

t<sub>HD</sub>


---

*The Highest Density (HD) interval of Student's t distribution.*


---

### Description

t<sub>HD</sub> is used to derive the narrowest interval of a predetermined coverage from a Student's t distribution, containing the Highest Density (HD) values.

**Usage**

```
t_HD( cover = NULL, df = NULL, mu = NULL, sdv = NULL, plot = FALSE,  
      xlab = "x", ylab = "Density" )
```

**Arguments**

cover	scalar (between 0 and 1); The coverage (probability) of the Highest Density (HD).
df	scalar (positive); It is the degrees of freedom of the Student's t distribution and needs to be a number.
mu	scalar (real); It is the mean of the Student's t distribution and needs to be a number. The default value is 0.
sdv	scalar (positive); It is the scale parameter of the Student's t distribution and needs to be a number. The default value is 1.
plot	logical; if TRUE, the plot of the HD region is displayed. It is FALSE by default.
xlab, ylab	The titles of the x-axis and y-axis for the HD plot. The default values are "x" and "Density".

**Details**

*t\_HD* provides the Highest Density (HD) of a Student's t distribution. HD is the narrowest interval with a pretermined coverage. The values included in HD have density larger than every point outside the region.

**Value**

Returns a data frame with the upper and the lower limits of the Highest Predictive Denisty or Mass (HPrD/M) region, the lower and the achieved coverage. If plot=T, then the HPrD/M is visualized.

**Examples**

```
t_HD( 0.95, df = 2, mu = 2, sdv = 3, plot = TRUE )
```

# Index

aPTT, [2](#)

betabinom\_HM, [3](#)

betanbinom\_HM, [4](#)

binom\_PCC, [5](#)

binom\_PRC, [7](#)

binom\_PRC\_h, [10](#)

compgamma\_HD, [12](#)

ECE, [13](#)

gamma\_PCC, [14](#)

gb2\_HD, [16](#)

invgamma\_PCC, [17](#)

lnorm1\_PCC, [19](#)

lnorm2\_PCC, [21](#)

lnorm3\_PCC, [23](#)

lnorm\_HD, [26](#)

lt\_HD, [27](#)

nbinom\_HM, [28](#)

nbinom\_PCC, [29](#)

norm1\_PCC, [31](#)

norm2\_PCC, [33](#)

norm3\_PCC, [35](#)

norm\_HD, [38](#)

norm\_mean2\_PRC, [39](#)

norm\_mean2\_PRC\_h, [42](#)

pois\_PCC, [43](#)

pois\_PRC, [46](#)

pois\_PRC\_h, [48](#)

t\_HD, [49](#)