Package ‘NonNorMvtDist’

October 12, 2022

Type Package

Title Multivariate Lomax (Pareto Type II) and Its Related Distributions

Version 1.0.2

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Description Implements calculation of probability density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for the following multivariate distributions: Lomax (Pareto Type II), generalized Lomax, Mardia’s Pareto of Type I, Logistic, Burr, Cook-Johnson’s uniform, F and Inverted Beta. See Tapan Nayak (1987) <doi:10.2307/3214068>.

Depends R (>= 3.6.0)

License GPL-3

Encoding UTF-8

LazyData true

Imports stats, cubature

RoxygenNote 7.0.2

NeedsCompilation no

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Repository CRAN

Date/Publication 2020-03-23 15:40:02 UTC

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`MvtBurr` *Multivariate Burr Distribution*

Description

Calculation of density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for multivariate Burr distribution with a scalar parameter `parm1` and vectors of parameters `parm2` and `parm3`.

Usage

```
dmvburr(x, parm1 = 1, parm2 = rep(1, k), parm3 = rep(1, k), log = FALSE)

pmvburr(q, parm1 = 1, parm2 = rep(1, k), parm3 = rep(1, k))

qmvburr(
  p,
  parm1 = 1,
  parm2 = rep(1, k),
  parm3 = rep(1, k),
  interval = c(0, 1e+08)
)

rmvburr(n, parm1 = 1, parm2 = rep(1, k), parm3 = rep(1, k))

smvburr(q, parm1 = 1, parm2 = rep(1, k), parm3 = rep(1, k))
```

Arguments

`x`	vector or matrix of quantiles. If x is a matrix, each row vector constitutes a vector of quantiles for which the density $f(x)$ is calculated (for i-th row x_i, $f(x_i)$ is reported).
`parm1`	a scalar parameter, see parameter a in **Details**.
`parm2`	a vector of parameters, see parameters d_i in **Details**.
`parm3`	a vector of parameters, see parameters c_i in **Details**.
`log`	logical; if TRUE, probability densities f are given as $log(f)$.
`q`	a vector of quantiles.
`p`	a scalar value corresponding to probability.
`interval`	a vector containing the end-points of the interval to be searched. Default value is set as `c(0, 1e8)`.
`n`	number of observations.
`k`	dimension of data or number of variates.

Details

Multivariate Burr distribution (Johnson and Kotz, 1972) is a joint distribution of positive random variables $X_1, \cdots, X_k$. Its probability density is given as

$$f(x_1, \cdots, x_k) = \frac{[\prod_{i=1}^{k} c_i d_i] a(a+1) \cdots (a+k-1) [\prod_{i=1}^{k} x_i^{c_i - 1}]}{(1 + \sum_{i=1}^{k} d_i x_i^{c_i})^{a+k}},$$

where $x_i > 0, a, c_i, d_i > 0, i = 1, \cdots, k$.

Cumulative distribution function $F(x_1, \ldots, x_k)$ is obtained by the following formula related to survival function $\bar{F}(x_1, \ldots, x_k)$ (Joe, 1997)

$$F(x_1, \ldots, x_k) = 1 + \sum_{S \in \mathcal{S}} (-1)^{|S|} \bar{F}_S(x_j, j \in S),$$

where the survival function is given by

$$\bar{F}(x_1, \cdots, x_k) = \left(1 + \sum_{i=1}^{k} d_i x_i^{c_i} \right)^{-a}.$$

Equicoordinate quantile is obtained by solving the following equation for q through the built-in one dimension root finding function `uniroot`:

$$\int_0^q \cdots \int_0^q f(x_1, \cdots, x_k) dx_k \cdots dx_1 = p,$$

where p is the given cumulative probability.

Random numbers $X_1, \cdots, X_k$ from multivariate Burr distribution can be generated through transformation of multivariate Lomax random variables $Y_1, \cdots, Y_k$ by letting $X_i = (\theta_i Y_i / d_i)^{1/c_i}, i = 1, \cdots, k$; see Nayak (1987).

Value

`dmvburr` gives the numerical values of the probability density.

`pmvburr` gives the cumulative probability.

`qmvburr` gives the equicoordinate quantile.

`rmvburr` generates random numbers.

`smvburr` gives the value of survival function.

References

Joe, H. (1997). *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Johnson, N. L. and Kotz, S. (1972). *Distribution in Statistics: Continuous Multivariate Distributions*. New York: John Wiley & Sons, INC.

Nayak, T. K. (1987). Multivariate Lomax Distribution: Properties and Usefulness in Reliability Theory. *Journal of Applied Probability*, Vol. 24, No. 1, 170-177.

See Also

uniroot for one dimensional root (zero) finding.

Examples

```
# Calculations for the multivariate Burr with parameters:
# a = 3, d1 = 1, d2 = 3, d3 = 5, c1 = 2, c2 = 4, c3 = 6
# Vector of quantiles: c(3, 2, 1)

dmvburr(x = c(3, 2, 1), parm1 = 3, parm2 = c(1, 3, 5), parm3 = c(2, 4, 6)) # Density

pmvburr(q = c(3, 2, 1), parm1 = 3, parm2 = c(1, 3, 5), parm3 = c(2, 4, 6)) # Cumulative Probability

# Equicoordinate quantile of cumulative probability 0.5
qmvburr(p = 0.5, parm1 = 3, parm2 = c(1, 3, 5), parm3 = c(2, 4, 6))

# Random numbers generation with sample size 100
rmvburr(n = 100, parm1 = 3, parm2 = c(1, 3, 5), parm3 = c(2, 4, 6))

smvburr(q = c(3, 2, 1), parm1 = 3, parm2 = c(1, 3, 5), parm3 = c(2, 4, 6)) # Survival function
```

MvtF *Multivariate F Distribution*

Description

Calculation of density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for multivariate F distribution with degrees of freedom df.

Usage

```
dmvf(x, df = rep(1, k + 1), log = FALSE)

pmvf(q, df = rep(1, k + 1), algorithm = c("numerical", "MC"), nsim = 1e+07)

qmvf(
  p,
  df = rep(1, k + 1),
  interval = c(1e-08, 1e+08),
  algorithm = c("numerical", "MC"),
  nsim = 1e+06
)

rmvf(n, df = rep(1, k + 1))

smvf(q, df = rep(1, k + 1), algorithm = c("numerical", "MC"), nsim = 1e+07)
```

Arguments

`x`	vector or matrix of quantiles. If x is a matrix, each row vector constitutes a vector of quantiles for which the density $f(x)$ is calculated (for i-th row x_i, $f(x_i)$ is reported).
`df`	a vector of $k+1$ degrees of freedom, see parameter $(2a, 2l_1, \dots, 2l_k)$ in **Details**.
`log`	logical; if TRUE, probability densities f are given as $log(f)$.
`q`	a vector of quantiles.
`algorithm`	method to be used for calculating cumulative probability. Two options are provided as (i) `numerical` using adaptive multivariate integral and (ii) `MC` using Monte Carlo method. Recommend algorithm `numerical` for $(k <= 4)$ dimension and `MC` for $(k > 4)$ dimension based on running time consumption. Default option is set as `numerical`.
`nsim`	number of simulations used in algorithm `MC`.
`p`	a scalar value corresponding to probability.
`interval`	a vector containing the end-points of the interval to be searched. Default value is set as `c(1e-8, 1e8)`.
`n`	number of observations.
`k`	dimension of data or number of variates.

Details

Multivariate F distribution (Johnson and Kotz, 1972) is a joint probability distribution of positive random variables and its probability density is given as

$$f(x_1, \cdots, x_k) = \frac{[\prod_{i=1}^k (l_i/a)^{l_i}]\Gamma(\sum_{i=1}^k l_i + a) \prod_{i=1}^k x_i^{l_i - 1}}{\Gamma(a)[\prod_{i=1}^k \Gamma(l_i)](1 + \sum_{i=1}^k \frac{l_i}{a} x_i)^{\sum_{i=1}^k l_i + a}},$$

where $x_i > 0, a > 0, l_i > 0, i = 1, \cdots, k$. The degrees of freedom are $(2a, 2l_1, \dots, 2l_k)$.

Cumulative distribution function $F(x_1, \dots, x_k)$ is obtained by multiple integral

$$F(x_1, \dots, x_k) = \int_0^{x_1} \cdots \int_0^{x_k} f(y_1, \cdots, y_k) dy_k \cdots dy_1.$$

This multiple integral is calculated by either adaptive multivariate integration using `hcubature` in package **cubature** (Narasimhan et al., 2018) or via Monte Carlo method.

Equicoordinate quantile is obtained by solving the following equation for q through the built-in one dimension root finding function `uniroot`:

$$\int_0^q \cdots \int_0^q f(x_1, \cdots, x_k) dx_k \cdots dx_1 = p,$$

where p is the given cumulative probability.

The survival function $\bar{F}(x_1, \cdots, x_k)$ is obtained either by the following formula related to cumulative distribution function $F(x_1, \dots, x_k)$ (Joe, 1997)

$$\bar{F}(x_1, \cdots, x_k) = 1 + \sum_{S \in \mathcal{S}} (-1)^{|S|} F_S(x_j, j \in S),$$

or via Monte Carlo method.

Random numbers $X_1, \cdots, X_k$ from multivariate F distribution can be generated through parameter substitutions in simulation of generalized multivariate Lomax distribution by letting $\theta_i = l_i/a, i = 1, \cdots, k$; see Nayak (1987).

Value

`dmvf` gives the numerical values of the probability density.

`pmvf` gives a list of two items:

`value` cumulative probability

`error` the estimated relative error by `algorithm = "numerical"` or the estimated standard error by `algorithm = "MC"`

`qmvf` gives the equicoordinate quantile. `NaN` is returned for no solution found in the given interval. The result is seed dependent if Monte Carlo algorithm is chosen (`algorithm = "MC"`).

`rmvf` generates random numbers.

`smvf` gives a list of two items:

`value` the value of survial function

`error` the estimated relative error by `algorithm = "numerical"` or the estimated standard error by `algorithm = "MC"`

References

Joe, H. (1997). *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Johnson, N. L. and Kotz, S. (1972). *Distribution in Statistics: Continuous Multivariate Distributions*. New York: John Wiley & Sons, INC.

Narasimhan, B., Koller, M., Johnson, S. G., Hahn, T., Bouvier, A., Kiêu, K. and Gaure, S. (2018). cubature: Adaptive Multivariate Integration over Hypercubes. R package version 2.0.3.

Nayak, T. K. (1987). Multivariate Lomax Distribution: Properties and Usefulness in Reliability Theory. *Journal of Applied Probability*, Vol. 24, No. 1, 170-177.

See Also

`uniroot` for one dimensional root (zero) finding.

Examples

```
# Calculations for the multivariate F with degrees of freedom:
# df = c(2, 4, 6)
# Vector of quantiles:  c(1, 2)

dmvf(x = c(1, 2), df = c(2, 4, 6)) # Density

# Cumulative Probability using adaptive multivariate integral
pmvf(q = c(1, 2), df = c(2, 4, 6), algorithm = "numerical")
```

```
# Cumulative Probability using Monte Carlo method
pmvf(q = c(1, 2), df = c(2, 4, 6), algorithm = "MC")


# Equicoordinate quantile of cumulative probability 0.5
qmvf(p = 0.5, df = c(2, 4, 6))

# Random numbers generation with sample size 100
rmvf(n = 100, df = c(2, 4, 6))

smvf(q = c(1, 2), df = c(2, 4, 6)) # Survival function
```

MvtGlomx *Generalized Multivariate Lomax (Pareto Type II) Distribution*

Description

Calculation of density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for generalized multivariate Lomax distribution with a scalar parameter `parm1` and vectors of parameters `parm2` and `parm3`.

Usage

```
dmvglomax(x, parm1 = 1, parm2 = rep(1, k), parm3 = rep(1, k), log = FALSE)

pmvglomax(
  q,
  parm1 = 1,
  parm2 = rep(1, k),
  parm3 = rep(1, k),
  algorithm = c("numerical", "MC"),
  nsim = 1e+07
)

qmvglomax(
  p,
  parm1 = 1,
  parm2 = rep(1, k),
  parm3 = rep(1, k),
  interval = c(1e-08, 1e+08),
  algorithm = c("numerical", "MC"),
  nsim = 1e+06
)

rmvglomax(n, parm1 = 1, parm2 = rep(1, k), parm3 = rep(1, k))

smvglomax(
```

```
  q,
  parm1 = 1,
  parm2 = rep(1, k),
  parm3 = rep(1, k),
  algorithm = c("numerical", "MC"),
  nsim = 1e+07
)
```

Arguments

`x`	vector or matrix of quantiles. If x is a matrix, each row vector constitutes a vector of quantiles for which the density $f(x)$ is calculated (for i-th row x_i, $f(x_i)$ is reported).
`parm1`	a scalar parameter, see parameter a in **Details**.
`parm2`	a vector of parameters, see parameters θ_i in **Details**.
`parm3`	a vector of parameters, see parameters l_i in **Details**.
`log`	logical; if TRUE, probability densities f are given as $log(f)$.
`q`	a vector of quantiles.
`algorithm`	method to be used for calculating cumulative probability. Two options are provided as (i) `numerical` using adaptive multivariate integral and (ii) `MC` using Monte Carlo method. Recommend algorithm `numerical` for $(k <= 4)$ dimension and `MC` for $(k > 4)$ dimension based on running time consumption. Default option is set as `numerical`.
`nsim`	number of simulations used in algorithm `MC`.
`p`	a scalar value corresponding to probability.
`interval`	a vector containing the end-points of the interval to be searched. Default value is set as `c(1e-8, 1e8)`.
`n`	number of observations.
`k`	dimension of data or number of variates.

Details

Generalized multivariate Lomax (Pareto type II) distribution was introduced by Nayak (1987) as a joint probability distribution of several skewed nonnegative random variables $X_1, X_2, \cdots, X_k$. Its probability density function is given by

$$f(x_1, \cdots, x_k) = \frac{[\prod_{i=1}^k \theta_i^{l_i}]\Gamma(\sum_{i=1}^k l_i + a)\prod_{i=1}^k x_i^{l_i-1}}{\Gamma(a)[\prod_{i=1}^k \Gamma(l_i)](1+\sum_{i=1}^k \theta_i x_i)^{\sum_{i=1}^k l_i + a}},$$

where $x_i > 0, a, \theta_i, l_i > 0, i = 1, \cdots, k$.

Cumulative distribution function $F(x_1, \ldots, x_k)$ is obtained by multiple integral

$$F(x_1, \ldots, x_k) = \int_0^{x_1} \cdots \int_0^{x_k} f(y_1, \cdots, y_k) dy_k \cdots dy_1.$$

This multiple integral is calculated by either adaptive multivariate integration using `hcubature` in package **cubature** (Narasimhan et al., 2018) or via Monte Carlo method.

Equicoordinate quantile is obtained by solving the following equation for q through the built-in one dimension root finding function uniroot:

$$\int_0^q \cdots \int_0^q f(x_1, \cdots, x_k) dx_k \cdots dx_1 = p,$$

where p is the given cumulative probability.

The survival function $\bar{F}(x_1, \cdots, x_k)$ is obtained either by the following formula related to cumulative distribution function $F(x_1, \ldots, x_k)$ (Joe, 1997)

$$\bar{F}(x_1, \cdots, x_k) = 1 + \sum_{S \in \mathcal{S}} (-1)^{|S|} F_S(x_j, j \in S),$$

or via Monte Carlo method.

Random numbers from generalized multivariate Lomax distribution can be generated by simulating k independent gamma random variables having a common parameter following gamma distribution with shape parameter a and scale parameter 1; see Nayak (1987).

Value

dmvglomax gives the numerical values of the probability density.

pmvglomax gives a list of two items:

value cumulative probability

error the estimated relative error by algorithm = "numerical" or the estimated standard error by algorithm = "MC"

qmvglomax gives the equicoordinate quantile. NaN is returned for no solution found in the given interval. The result is seed dependent if Monte Carlo algorithm is chosen (algorithm = "MC").

rmvglomax generates random numbers.

smvglomax gives a list of two items:

value the value of survial function

error the estimated relative error by algorithm = "numerical" or the estimated standard error by algorithm = "MC"

References

Joe, H. (1997). *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Narasimhan, B., Koller, M., Johnson, S. G., Hahn, T., Bouvier, A., Kiêu, K. and Gaure, S. (2018). cubature: Adaptive Multivariate Integration over Hypercubes. R package version 2.0.3.

Nayak, T. K. (1987). Multivariate Lomax Distribution: Properties and Usefulness in Reliability Theory. *Journal of Applied Probability*, Vol. 24, No. 1, 170-177.

See Also

uniroot for one dimensional root (zero) finding.

Examples

```
# Calculations for the generalized multivariate Lomax with parameters:
# a = 5, theta1 = 1, theta2 = 2, l1 = 4, l2 = 5
# Vector of quantiles: c(5, 6)

dmvglomax(x = c(5, 6), parm1 = 5, parm2 = c(1, 2), parm3 = c(4, 5)) # Density


# Cumulative Probability using adaptive multivariate integral
pmvglomax(q = c(5, 6), parm1 = 5, parm2 = c(1, 2), parm3 = c(4, 5))


# Cumulative Probability using Monte Carlo method
pmvglomax(q = c(5, 6), parm1 = 5, parm2 = c(1, 2), parm3 = c(4, 5), algorithm = "MC")


# Equicoordinate quantile of cumulative probability 0.5
qmvglomax(p = 0.5, parm1 = 5, parm2 = c(1, 2), parm3 = c(4, 5))

# Random numbers generation with sample size 100
rmvglomax(n = 100, parm1 = 5, parm2 = c(1, 2), parm3 = c(4, 5))

smvglomax(q = c(5, 6), parm1 = 5, parm2 = c(1, 2), parm3 = c(4, 5)) # Survival function
```

MvtInvBeta *Multivariate Inverted Beta Distribution*

Description

Calculation of density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for multivariate inverted beta distribution with a scalar parameter `parm1` and a vector of parameters `parm2`.

Usage

```
dmvinvbeta(x, parm1 = 1, parm2 = rep(1, k), log = FALSE)

pmvinvbeta(
  q,
  parm1 = 1,
  parm2 = rep(1, k),
  algorithm = c("numerical", "MC"),
  nsim = 1e+07
)

qmvinvbeta(
  p,
```

```
  parm1 = 1,
  parm2 = rep(1, k),
  interval = c(1e-08, 1e+08),
  algorithm = c("numerical", "MC"),
  nsim = 1e+06
)

rmvinvbeta(n, parm1 = 1, parm2 = rep(1, k))

smvinvbeta(
  q,
  parm1 = 1,
  parm2 = rep(1, k),
  algorithm = c("numerical", "MC"),
  nsim = 1e+07
)
```

Arguments

`x`	vector or matrix of quantiles. If x is a matrix, each row vector constitutes a vector of quantiles for which the density $f(x)$ is calculated (for i-th row x_i, $f(x_i)$ is reported).
`parm1`	a scalar parameter, see parameter a in **Details**.
`parm2`	a vector of parameters, see parameter l_i in **Details**.
`log`	logical; if TRUE, probability densities f are given as $log(f)$.
`q`	a vector of quantiles.
`algorithm`	method to be used for calculating cumulative probability. Two options are provided as (i) `numerical` using adaptive multivariate integral and (ii) `MC` using Monte Carlo method. Recommend algorithm `numerical` for $(k <= 4)$ dimension and `MC` for $(k > 4)$ dimension based on running time consumption. Default option is set as `numerical`.
`nsim`	number of simulations used in algorithm `MC`.
`p`	a scalar value corresponding to probability.
`interval`	a vector containing the end-points of the interval to be searched. Default value is set as `c(1e-8, 1e8)`.
`n`	number of observations.
`k`	dimension of data or number of variates.

Details

Multivariate inverted beta distribution is an alternative expression of multivariate F distribution and is a special case of multivariate Lomax distribution (Balakrishnan and Lai, 2009). Its probability density is given as

$$f(x_1, \cdots, x_p) = \frac{\Gamma(\sum_{i=1}^p l_i + a) \prod_{i=1}^p x_i^{l_i - 1}}{\Gamma(a)[\prod_{i=1}^p \Gamma(l_i)](1 + \sum_{i=1}^p x_i)^{\sum_{i=1}^p l_i + a}},$$

where $x_i > 0, a > 0, l_i > 0, i = 1, \cdots, p$.

Cumulative distribution function $F(x_1, \ldots, x_k)$ is obtained by multiple integral

$$F(x_1, \ldots, x_k) = \int_0^{x_1} \cdots \int_0^{x_k} f(y_1, \cdots, y_k) dy_k \cdots dy_1.$$

This multiple integral is calculated by either adaptive multivariate integration using hcubature in package **cubature** (Narasimhan et al., 2018) or via Monte Carlo method.

Equicoordinate quantile is obtained by solving the following equation for q through the built-in one dimension root finding function uniroot:

$$\int_0^q \cdots \int_0^q f(x_1, \cdots, x_k) dx_k \cdots dx_1 = p,$$

where p is the given cumulative probability.

The survival function $\bar{F}(x_1, \cdots, x_k)$ is obtained either by the following formula related to cumulative distribution function $F(x_1, \ldots, x_k)$ (Joe, 1997)

$$\bar{F}(x_1, \cdots, x_k) = 1 + \sum_{S \in \mathcal{S}} (-1)^{|S|} F_S(x_j, j \in S),$$

or via Monte Carlo method.

Random numbers $X_1, \cdots, X_k$ from multivariate inverted beta distribution can be generated through parameter substitutions in simulation of generalized multivariate Lomax distribution by letting $\theta_i = 1, i = 1, \cdots, k$.

Value

dmvinvbeta gives the numerical values of the probability density.

pmvinvbeta gives a list of two items:

value cumulative probability

error the estimated relative error by algorithm = "numerical" or the estimated standard error by algorithm = "MC"

qmvinvbeta gives the equicoordinate quantile. NaN is returned for no solution found in the given interval. The result is seed dependent if Monte Carlo algorithm is chosen (algorithm = "MC").

rmvinvbeta generates random numbers.

smvinvbeta gives a list of two items:

value the value of survial function

error the estimated relative error by algorithm = "numerical" or the estimated standard error by algorithm = "MC"

References

Balakrishnan, N. and Lai, C. (2009). *Continuous Bivariate Distributions. 2nd Edition.* New York: Springer.

Joe, H. (1997). *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Narasimhan, B., Koller, M., Johnson, S. G., Hahn, T., Bouvier, A., Kiêu, K. and Gaure, S. (2018). cubature: Adaptive Multivariate Integration over Hypercubes. R package version 2.0.3.

Nayak, T. K. (1987). Multivariate Lomax Distribution: Properties and Usefulness in Reliability Theory. *Journal of Applied Probability*, Vol. 24, No. 1, 170-177.

See Also

uniroot for one dimensional root (zero) finding.

Examples

```
# Calculations for the multivariate inverted beta with parameters:
# a = 7, l1 = 1, l2 = 3
# Vector of quantiles: c(2, 4)

dmvinvbeta(x = c(2, 4), parm1 = 7, parm2 = c(1, 3)) # Density

# Cumulative Probability using adaptive multivariate integral
pmvinvbeta(q = c(2, 4), parm1 = 7, parm2 = c(1, 3))


# Cumulative Probability using Monte Carlo method
pmvinvbeta(q = c(2, 4), parm1 = 7, parm2 = c(1, 3), algorithm = "MC")


# Equicoordinate quantile of cumulative probability 0.5
qmvinvbeta(p = 0.5, parm1 = 7, parm2 = c(1, 3))

# Random numbers generation with sample size 100
rmvinvbeta(n = 100, parm1 = 7, parm2 = c(1, 3))

smvinvbeta(q = c(2, 4), parm1 = 7, parm2 = c(1, 3)) # Survival function
```

MvtLogis *Multivariate Logistic Distribution*

Description

Calculation of density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for multivariate logistic distribution with vector parameter parm1 and vector parameter parm2.

Usage

```
dmvlogis(x, parm1 = rep(1, k), parm2 = rep(1, k), log = FALSE)

pmvlogis(q, parm1 = rep(1, k), parm2 = rep(1, k))
```

```
qmvlogis(p, parm1 = rep(1, k), parm2 = rep(1, k), interval = c(0, 1e+08))

rmvlogis(n, parm1 = rep(1, k), parm2 = rep(1, k))

smvlogis(q, parm1 = rep(1, k), parm2 = rep(1, k))
```

Arguments

`x`	vector or matrix of quantiles. If x is a matrix, each row vector constitutes a vector of quantiles for which the density $f(x)$ is calculated (for i-th row x_i, $f(x_i)$ is reported).
`parm1`	a vector of location parameters, see parameter μ_i in **Details**.
`parm2`	a vector of scale parameters, see parameters σ_i in **Details**.
`log`	logical; if TRUE, probability densities f are given as $log(f)$.
`q`	a vector of quantiles.
`p`	a scalar value corresponding to probability.
`interval`	a vector containing the end-points of the interval to be searched. Default value is set as `c(0, 1e8)`.
`n`	number of observations.
`k`	dimension of data or number of variates.

Details

Bivariate logistic distribution was introduced by Gumbel (1961) and its multivariate generalization was given by Malik and Abraham (1973) as

$$f(x_1, \cdots, x_k) = \frac{k! \exp\left(-\sum_{i=1}^{k} \frac{x_i - \mu_i}{\sigma_i}\right)}{[\prod_{i=1}^{p} \sigma_i][1 + \sum_{i=1}^{k} \exp\left(-\frac{x_i - \mu_i}{\sigma_i}\right)]^{1+k}},$$

where $-\infty < x_i, \mu_i < \infty, \sigma_i > 0, i = 1, \cdots, k.$

Cumulative distribution function $F(x_1, \dots, x_k)$ is given as

$$F(x_1, \cdots, x_k) = \left[1 + \sum_{i=1}^{k} \exp(-\frac{x_i - \mu_i}{\sigma_i})\right]^{-1}.$$

Equicoordinate quantile is obtained by solving the following equation for q through the built-in one dimension root finding function `uniroot`:

$$\int_{-\infty}^{q} \cdots \int_{-\infty}^{q} f(x_1, \cdots, x_k) dx_k \cdots dx_1 = p,$$

where p is the given cumulative probability.

The survival function $\bar{F}(x_1, \cdots, x_k)$ is obtained by the following formula related to cumulative distribution function $F(x_1, \dots, x_k)$ (Joe, 1997)

$$\bar{F}(x_1, \cdots, x_k) = 1 + \sum_{S \in \mathcal{S}} (-1)^{|S|} F_S(x_j, j \in S).$$

Random numbers $X_1, \cdots, X_k$ from multivariate logistic distribution can be generated through transformation of multivariate Lomax random variables $Y_1, \cdots, Y_k$ by letting $X_i = \mu_i - \sigma_i \ln(\theta_i Y_i), i = 1, \cdots, k$; see Nayak (1987).

Value

`dmvlogis` gives the numerical values of the probability density.

`pmvlogis` gives the cumulative probability.

`qmvlogis` gives the equicoordinate quantile.

`rmvlogis` generates random numbers.

`smvlogis` gives the value of survival function

References

Gumbel, E.J. (1961). Bivariate logistic distribution. *J. Am. Stat. Assoc.*, 56, 335-349.

Joe, H. (1997). *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Malik, H. J. and Abraham, B. (1973). Multivariate logistic distributions. *Ann. Statist.* 3, 588-590.

Nayak, T. K. (1987). Multivariate Lomax Distribution: Properties and Usefulness in Reliability Theory. *Journal of Applied Probability*, Vol. 24, No. 1, 170-177.

See Also

`uniroot` for one dimensional root (zero) finding.

Examples

```
# Calculations for the multivariate logistic distribution with parameters:
# mu1 = 0.5, mu2 = 1, mu3 = 2, sigma1 = 1, sigma2 = 2 and sigma3 = 3
# Vector of quantiles: c(3, 2, 1)

dmvlogis(x = c(3, 2, 1), parm1 = c(0.5, 1, 2), parm2 = c(1, 2, 3)) # Density

pmvlogis(q = c(3, 2, 1), parm1 = c(0.5, 1, 2), parm2 = c(1, 2, 3)) # Cumulative Probability

# Equicoordinate quantile of cumulative probability 0.5
qmvlogis(p = 0.5, parm1 = c(0.5, 1, 2), parm2 = c(1, 2, 3))

# Random numbers generation with sample size 100
rmvlogis(n = 100, parm1 = c(0.5, 1, 2), parm2 = c(1, 2, 3))

smvlogis(q = c(3, 2, 1), parm1 = c(0.5, 1, 2), parm2 = c(1, 2, 3)) # Survival function
```

MvtLomax *Multivariate Lomax (Pareto Type II) Distribution*

Description

Calculation of density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for multivariate Lomax (Pareto Type II) distribution with a scalar parameter `parm1` and vector parameter `parm2`.

Usage

```
dmvlomax(x, parm1 = 1, parm2 = rep(1, k), log = FALSE)

pmvlomax(q, parm1 = 1, parm2 = rep(1, k))

qmvlomax(p, parm1 = 1, parm2 = rep(1, k), interval = c(0, 1e+08))

rmvlomax(n, parm1 = 1, parm2 = rep(1, k))

smvlomax(q, parm1 = 1, parm2 = rep(1, k))
```

Arguments

`x`	vector or matrix of quantiles. If x is a matrix, each row vector constitutes a vector of quantiles for which the density $f(x)$ is calculated (for i-th row x_i, $f(x_i)$ is reported).
`parm1`	a scalar parameter, see parameter a in **Details**.
`parm2`	a vector of parameters, see parameters θ_i in **Details**.
`log`	logical; if TRUE, probability densities f are given as $log(f)$.
`q`	a vector of quantiles.
`p`	a scalar value corresponding to probability.
`interval`	a vector containing the end-points of the interval to be searched. Default value is set as `c(0, 1e8)`.
`n`	number of observations.
`k`	dimension of data or number of variates.

Details

Multivariate Lomax (Pareto type II) distribution was introduced by Nayak (1987) as a joint probability distribution of several skewed positive random variables $X_1, X_2, \cdots, X_k$. Its probability density function is given by

$$f(x_1, x_2, \ldots, x_k) = \frac{[\prod_{i=1}^{k} \theta_i] a(a+1)\cdots(a+k-1)}{(1+\sum_{i=1}^{k} \theta_i x_i)^{a+k}},$$

where $x_i > 0, a > 0, \theta_i > 0, i = 1, \ldots, k$.

Cumulative distribution function $F(x_1, \ldots, x_k)$ is obtained by the following formula related to survival function $\bar{F}(x_1, \ldots, x_k)$ (Joe, 1997)

$$F(x_1, \ldots, x_k) = 1 + \sum_{S \in \mathcal{S}} (-1)^{|S|} \bar{F}_S(x_j, j \in S),$$

where the survival function is given by

$$\bar{F}(x_1, \ldots, x_k) = (1 + \sum_{i=1}^{k} \theta_i x_i)^{-a}.$$

Equicoordinate quantile is obtained by solving the following equation for q through the built-in one dimension root finding function uniroot:

$$\int_0^q \cdots \int_0^q f(x_1, \cdots, x_k) dx_k \cdots dx_1 = p,$$

where p is the given cumulative probability.

Random numbers from multivariate Lomax distribution can be generated by simulating k independent exponential random variables having a common environment parameter following gamma distribution with shape parameter a and scale parameter 1; see Nayak (1987).

Value

`dmvlomax` gives the numerical values of the probability density.

`pmvlomax` gives the cumulative probability.

`qmvlomax` gives the equicoordinate quantile.

`rmvlomax` generates random numbers.

`smvlomax` gives the value of survival function.

References

Joe, H. (1997). *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Nayak, T. K. (1987). Multivariate Lomax Distribution: Properties and Usefulness in Reliability Theory. *Journal of Applied Probability*, Vol. 24, No. 1, 170-177.

See Also

`uniroot` for one dimensional root (zero) finding.

Examples

```
# Calculations for the multivariate Lomax with parameters:
# a = 5, theta1 = 1, theta2 = 2 and theta3 = 3.
# Vector of quantiles: c(3, 2, 1)

dmvlomax(x = c(3, 2, 1), parm1 = 5, parm2 = c(1, 2, 3)) # Density
```

```
pmvlomax(q = c(3, 2, 1), parm1 = 5, parm2 = c(1, 2, 3)) # Cumulative Probability

# Equicoordinate quantile of cumulative probability 0.5
qmvlomax(p = 0.5, parm1 = 5, parm2 = c(1, 2, 3))

# Random numbers generation with sample size 100
rmvlomax(n = 100, parm1 = 5, parm2 = c(1, 2, 3))

smvlomax(q = c(3, 2, 1), parm1 = 5, parm2 = c(1, 2, 3)) # Survival function
```

MvtMardiaPareto1 *Mardia's Multivariate Pareto Type I Distribution*

Description

Calculation of density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for Mardia's multivariate Pareto Type I distribution with a scalar parameter parm1 and a vector of parameters parm2.

Usage

```
dmvmpareto1(x, parm1 = 1, parm2 = rep(1, k), log = FALSE)

pmvmpareto1(q, parm1 = 1, parm2 = rep(1, k))

qmvmpareto1(
  p,
  parm1 = 1,
  parm2 = rep(1, k),
  interval = c(max(1/parm2) + 1e-08, 1e+08)
)

rmvmpareto1(n, parm1 = 1, parm2 = rep(1, k))

smvmpareto1(q, parm1 = 1, parm2 = rep(1, k))
```

Arguments

x	vector or matrix of quantiles. If x is a matrix, each row vector constitutes a vector of quantiles for which the density $f(x)$ is calculated (for i-th row x_i, $f(x_i)$ is reported).
parm1	a scalar parameter, see parameter a in **Details**.
parm2	a vector of parameters, see parameters θ_i in **Details**.
log	logical; if TRUE, probability densities f are given as $log(f)$.
q	a vector of quantiles.

`p`	a scalar value corresponding to probability.
`interval`	a vector containing the end-points of the interval to be searched. Default value is set as `c(max(1 / parm2) + 1e-8, 1e8)` according to $x_i > 1/\theta_i, \theta_i > 0, i = 1, \cdots, k$.
`n`	number of observations.
`k`	dimension of data or number of variates.

Details

Multivariate Pareto type I distribution was introduced by Mardia (1962) as a joint probability distribution of several nonnegative random variables $X_1, \cdots, X_k$. Its probability density function is given by

$$f(x_1, \cdots, x_k) = \frac{[\prod_{i=1}^k \theta_i] a(a+1)\cdots(a+k-1)}{(\sum_{i=1}^k \theta_i x_i - k + 1)^{a+k}},$$

where $x_i > 1/\theta_i, a > 0, \theta_i > 0, i = 1, \cdots, k$.

Cumulative distribution function $F(x_1, \ldots, x_k)$ is obtained by the following formula related to survival function $\bar{F}(x_1, \ldots, x_k)$ (Joe, 1997)

$$F(x_1, \ldots, x_k) = 1 + \sum_{S \in \mathcal{S}} (-1)^{|S|} \bar{F}_S(x_j, j \in S),$$

where the survival function is given by

$$\bar{F}(x_1, \cdots, x_k) = \left(\sum_{i=1}^k \theta_i x_i - k + 1 \right)^{-a}.$$

Equicoordinate quantile is obtained by solving the following equation for q through the built-in one dimension root finding function `uniroot`:

$$\int_0^q \cdots \int_0^q f(x_1, \cdots, x_k) dx_k \cdots dx_1 = p,$$

where p is the given cumulative probability.

Random numbers $X_1, \cdots, X_k$ from Mardia's multivariate Pareto type I distribution can be generated through linear transformation of multivariate Lomax random variables $Y_1, \cdots, Y_k$ by letting $X_i = Y_i + 1/\theta_i, i = 1, \cdots, k$; see Nayak (1987).

Value

`dmvmpareto1` gives the numerical values of the probability density.

`pmvmpareto1` gives the cumulative probability.

`qmvmpareto1` gives the equicoordinate quantile.

`rmvmpareto1` generates random numbers.

`smvmpareto1` gives the value of survival function.

References

Joe, H. (1997). *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Mardia, K. V. (1962). Multivariate Pareto distributions. *Ann. Math. Statist.* 33, 1008-1015.

Nayak, T. K. (1987). Multivariate Lomax Distribution: Properties and Usefulness in Reliability Theory. *Journal of Applied Probability*, Vol. 24, No. 1, 170-177.

See Also

`uniroot` for one dimensional root (zero) finding.

Examples

```
# Calculations for the Mardia's multivariate Pareto Type I with parameters:
# a = 5, theta1 = 1, theta2 = 2, theta3 = 3
# Vector of quantiles: c(2, 1, 1)

dmvmpareto1(x = c(2, 1, 1), parm1 = 5, parm2 = c(1, 2, 3)) # Density

pmvmpareto1(q = c(2, 1, 1), parm1 = 5, parm2 = c(1, 2, 3)) # Cumulative Probability

# Equicoordinate quantile of cumulative probability 0.5
qmvmpareto1(p = 0.5, parm1 = 5, parm2 =  c(1, 2, 3))

# Random numbers generation with sample size 100
rmvmpareto1(n = 100, parm1 = 5, parm2 = c(1, 2, 3))

smvmpareto1(q = c(2, 1, 1), parm1 = 5, parm2 = c(1, 2, 3)) # Survival function
```

MvtUniform *Cook-Johnson's Multivariate Uniform Distribution*

Description

Calculation of density function, cumulative distribution function, equicoordinate quantile function and survival function, and random numbers generation for Cook-Johnson's multivariate uniform distribution with a scalar parameter `parm`.

Usage

```
dmvunif(x, parm = 1, log = FALSE)

pmvunif(q, parm = 1)

qmvunif(p, parm = 1, dim = k, interval = c(0, 1))

rmvunif(n, parm = 1, dim = 1)

smvunif(q, parm = 1)
```

Arguments

`x`	vector or matrix of quantiles. If x is a matrix, each row vector constitutes a vector of quantiles for which the density $f(x)$ is calculated (for i-th row x_i, $f(x_i)$ is reported).
`parm`	a scalar parameter, see parameter a in **Details**.
`log`	logical; if TRUE, probability densities f are given as $log(f)$.
`q`	a vector of quantiles.
`p`	a scalar value corresponding to probability.
`dim`	dimension of data or number of variates (k).
`interval`	a vector containing the end-points of the interval to be searched. Default value is set as `c(0, 1)`.
`n`	number of observations.

Details

Multivariate uniform distribution of Cook and Johnson (1981) is a joint distribution of uniform variables over $(0, 1]$ and its probability density is given as

$$f(x_1, \cdots, x_k) = \frac{\Gamma(a+k)}{\Gamma(a)a^k} \prod_{i=1}^{k} x_i^{(-1/a)-1} \left[\sum_{i=1}^{k} x_i^{-1/a} - k + 1 \right]^{-(a+k)},$$

where $0 < x_i <= 1, a > 0, i = 1, \cdots, k$. In fact, Cook-Johnson's uniform distribution is also called Clayton copula (Nelsen, 2006).

Cumulative distribution function $F(x_1, \ldots, x_k)$ is given as

$$F(x_1, \cdots, x_k) = \left[\sum_{i=1}^{k} x_i^{-1/a} - k + 1 \right]^{-a}.$$

Equicoordinate quantile is obtained by solving the following equation for q through the built-in one dimension root finding function `uniroot`:

$$\int_0^q \cdots \int_0^q f(x_1, \cdots, x_k) dx_k \cdots dx_1 = p,$$

where p is the given cumulative probability.

The survival function $\bar{F}(x_1, \cdots, x_k)$ is obtained by the following formula related to cumulative distribution function $F(x_1, \ldots, x_k)$ (Joe, 1997)

$$\bar{F}(x_1, \cdots, x_k) = 1 + \sum_{S \in \mathcal{S}} (-1)^{|S|} F_S(x_j, j \in S).$$

Random numbers $X_1, \cdots, X_k$ from Cook-Johnson's multivariate uniform distribution can be generated through transformation of multivariate Lomax random variables $Y_1, \cdots, Y_k$ by letting $X_i = (1 + \theta_i Y_i)^{-a}, i = 1, \cdots, k$; see Nayak (1987).

Value

dmvunif gives the numerical values of the probability density.

pmvunif gives the cumulative probability.

qmvunif gives the equicoordinate quantile.

rmvunif generates random numbers.

smvunif gives the value of survival function.

References

Cook, R. E. and Johnson, M. E. (1981). A family of distributions for modeling non-elliptically symmetric multivariate data. *J.R. Statist. Soc*. B 43, No. 2, 210-218.

Joe, H. (1997). *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Nayak, T. K. (1987). Multivariate Lomax Distribution: Properties and Usefulness in Reliability Theory. *Journal of Applied Probability*, Vol. 24, No. 1, 170-177.

Nelsen, R. B. (2006). *An Introduction to Copulas, Second Edition*. New York: Springer.

See Also

uniroot for one dimensional root (zero) finding.

Examples

```
# Calculations for the Cook-Johnson's multivariate uniform distribution with parameters:
# a = 2, dim = 3
# Vector of quantiles: c(0.8, 0.5, 0.2)

dmvunif(x = c(0.8, 0.5, 0.2), parm = 2) # Density

pmvunif(q = c(0.8, 0.5, 0.2), parm = 2) # Cumulative Probability

# Equicoordinate quantile of cumulative probability 0.5
qmvunif(p = 0.5, parm = 2, dim = 3)

# Random numbers generation with sample size 100
rmvunif(n = 100, parm = 2, dim = 3)

smvunif(q = c(0.8, 0.5, 0.2), parm = 3) # Survival function
```

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